A Theory of Intra-Party Factions and Electoral Accountability

Michael Pomirchy*

Word Count: 9,065

Abstract

Intra-party factions have increasingly wielded influence on key agenda items in the U.S. Congress. To better understand the welfare implications of these groups, this paper presents a formal model in which incumbents choose to join a faction and cast a vote on the majority party's agenda, after which an election takes place. This theory shows that while factions may enable incumbents to vote against the party's agenda, factions can have positive effects on party welfare by signaling incumbents' ideological type to their districts, thereby improving incumbents' reelection prospects and increasing the number of seats held by the party. I present case studies on the House Freedom Caucus's opposition to a funding measure for the DHS and the Blue Dog Democrats' defections on cap-and-trade to illustrate the theory.

*Postdoctoral Research Associate, Institution for Social and Policy Studies, Yale University.

Intra-party factions have garnered widespread attention in the press for their growing size and influence in the U.S. Congress. In the 115th Congress, almost all incoming legislators had joined an intra-party faction (Thomsen, 2017), which can be defined as a member organization situated outside the formal committee and party structures to achieve certain policy aims (Hammond, 2001). In part because of their growing membership, factions like the House Freedom Caucus (HFC) and the Congressional Progressive Caucus (CPC) have played a key role on important roll-call votes, such as when the HFC voted against Speaker Boehner's proposed funding measure for the Department of Homeland Security in 2015,[1] or when over three-quarters of the CPC opposed Speaker Pelosi's effort to send humanitarian aid to children who were separated from their families at the border in 2019.[2] However, the scholarly study of intra-party factions is still at a fairly nascent stage.

Some recent work in this area has made important advances and identified at least two roles that factions play in the U.S. Congress: (1) factions provide benefits and resources to help offset the costs of reprisal from party leadership when members fail to toe the party line (Bloch Rubin, 2017); and (2) intra-party factions create a specific electoral brand for members of Congress (Clarke, 2020). The first role entails enabling dissident members of Congress to achieve their policy aims. Individual legislators may lose committee assignments or campaign funds if they deviate from the party line (Patty, 2008; Pearson, 2015), so institutional organizations like factions help to mitigate these losses and liberate their members to vote more freely (Bloch Rubin, 2013; Clarke, n.d.). In this sense, factions protect incumbents from the pressures they may face from strong party leaders.

The second role is clear in how members market themselves in congressional newsletters and in the mass media, as Clarke (2020) points out. For example, one message from Rep. Raul Labrador (R-ID) in a congressional newsletter reads "[the] aim [of the House Freedom Caucus] is to advance an agenda of a limited, constitutional government and give voice

[1] *Roll Call*, "House Agrees to Fund Homeland Security Department," March 3, 2015.
[2] CNN, "House passes border funding bill after Pelosi reversal outrages progressives," June 27, 2019.

to those who feel Washington doesn't represent them."[3] Moreover, "we expect our group to grow, comprising a block of votes large enough to help shift the House to a more conservative footing." Another message from Rep. Mike Ross (D-AR) reads "as an active member and leader of the fiscally conservative, Democratic Blue Dog Coalition, I have been sounding the alarms on our national debt for years."[4] In each case, ideology is explicitly invoked through the representative's role in an intra-party faction.

While this gives us a better understanding of the role factions play, their welfare implications are still relatively unclear. This paper proposes a formal model that suggests that U.S. intra-party factions may enhance party welfare, particularly when parties care about reelecting their members. This yields different implications than more pessimistic depictions of factionalism in the public discourse, in which factions are perceived as precipitating conflict and promoting division within their party's ranks (Boucek, 2009). Historically, these perceptions date back to 17th- and 18th-century writers who viewed factions as "either contrary to the public spirit or as obstacles (albeit inevitable) to majority rule" (Boucek, 2009; page 460). In particular, Madison (1788) famously decried the "mischiefs of faction" and argued that their disruptive capabilities were dangerous for democratic government. Even today, as Dewan and Squintani (2015) note, "[factions] are commonly perceived as playing a negative role in political life" (page 875).

This paper argues that while factions may encourage incumbents to vote against their party on important roll-call votes, factions can also be a boon for parties by signaling incumbents' ideological type to their districts, thereby improving incumbents' reelection prospects and increasing the number of seats held by the party. While voters may already perceive Republicans as right-leaning and Democrats as left-leaning, intra-party labels deliver greater information that voters can use to identify whether their incumbent is ideologically congruent. As such, in equilibrium, faction membership can be part of a sufficient condition for

[3]Rep. Raul Labrador, Title: "A freedom agenda, from the bottom up," January 29th, 2015.

[4]Rep. Mike Ross, Title: "Legislative Update: Responsible Spending Cuts Key to Deficit Reduction," February 20th, 2011.

incumbents to win reelection in their district. If parties care about increasing their share of seats and maintaining majority status, factions can ultimately be beneficial to parties, even if faction members seek to obstruct the party's agenda. As a result, parties intent on reelecting their members may prefer to preserve dissident factions instead of undermine them.

More generally, one novel feature of the present model, in contrast to other game-theoretic accounts of factions, is that it examines faction membership in an accountability framework. Prior formal theory literature on factions either does not model voters at all (e.g., Dewan and Squintani, 2015; Laver and Shepsle, 1990) or does not conceptualize a legislator's faction affiliation as an observable signal to voters (Invernizzi, 2023). The theory I present, on the other hand, posits that legislators' decisions to join a faction bear electoral consequences, and this in turn affects membership decisions. Moreover, in the Appendix, I derive the optimal cost of party discipline and endogenous faction platforms. I present parties' optimal strategies for imposing costs on defections on roll-call votes and solve for equilibrium faction platforms as a function of the ideal points of the faction members.

In the first stage of the model, incumbents decide to join a faction within their party and then vote on the majority party's agenda. Afterwards, the voters in each district either reelect the incumbent or elect a random challenger. In this game, there are two main ingredients, which relate to the aforementioned work on factions (Bloch Rubin, 2017; Clarke, 2020). The first ingredient is the assumption that factions provide resources to mitigate the cost of party discipline. The second ingredient is that voters are uncertain about the precise ideological preferences of their incumbent (i.e., how liberal or conservative they are). As a result, voters update their beliefs about the incumbent based on the incumbent's roll-call votes and faction membership.

After detailing the results, I present two case studies to illustrate the theory. I first discuss an ideologically extreme Republican faction, the House Freedom Caucus, and its opposition to a funding measure of the Department of Homeland Security (DHS) in 2015. I

then discuss a moderate Democratic faction, the Blue Dog Coalition, in the 111th Congress and their role in the passage of Waxman-Markey, a bill that implements cap-and-trade.

Contributions to Literature

The parties literature traditionally conceives of American political parties as providing a durable public good for members of Congress that solves an internal collective action problem (e.g., Aldrich, 1995; Cox and McCubbins, 1993, 2005). Incumbents delegate power to the party leadership, where they can set the agenda for legislators to vote on. To ensure a cohesive bloc on these votes, parties discipline their members when they defect (Patty, 2008; Pearson, 2015). In addition, parties cultivate an electoral brand/reputation (Snyder and Ting, 2002) and promote incumbents to leadership positions (Patty et al., 2019), which can help party members when they go up for reelection. Often, the reputation of the party depends on its legislative record of accomplishment (Cox and McCubbins, 2005); as such, given the intensity of political competition in the modern era (Lee, 2016), the opposition party has an incentive to distance themselves from the other party and engage in conflict, not necessarily on ideological lines, but to make the competence of the reining party look weak (Gilmour, 1995; Lee, 2009).[5]

Recent work has applied many of these foundational concepts, particularly the idea of brands and collective action, to the study of intra-party legislative behavior. Like parties, intra-party groups solve a collective action problem in Congress, where absent the public good they provide, individuals are incentivized to free-ride on others to achieve policy gains (Bloch Rubin, 2017). They have leadership structures that organize meetings and provide a forum to deliberate and discuss ongoing legislation (Hammond, 2001). Finally, factions help

[5]More recently, there have been some theories that conceive of parties as a coalition of interest groups and activists (Bawn et al., 2012; Cohen et al., 2008; McCarty and Schickler, 2018). Other theories in the literature suggest that party leadership will be more powerful when the party is united; if there is significant homogeneity within the parties and significant heterogeneity between the two parties, we would see noncentrist outcomes (Aldrich, 1995; Rohde, 1991).

their members by fundraising for them through political action committees and by facilitating a sub-party brand (Clarke, 2020). Much like these works have applied canonical notions in the parties literature to intra-party behavior, the present analysis begins by applying models of party brands (e.g., Snyder and Ting, 2002) to the sub-party context. In particular, the theory presented here formally describes how legislators can further delineate their ideological type, beyond their party's reputation, through intra-party labels.

Generally, prior work has made empirical advances in this area on two major fronts: (1) the benefits of faction membership; and (2) the growth of extremist factions. In the first strand of literature, empirical work has shown that joining a faction can impact a legislator's donor base (Clarke, 2020). In particular, ideologically extreme factions have increased their membership in recent years (Thomsen, 2017), which can at least partially be explained by the dependence on access to certain campaign finance networks (Hendry and Sin, 2014). This paper makes a contribution by offering empirical predictions on faction membership and electoral prospects. First, legislators who join a faction that is ideologically congruent with their constituency's preferences are more likely to be reelected. Secondly, because of these electoral rewards/pressures, in equilibrium, faction members represent districts that are ideologically distinct from those represented by their co-partisans. Lastly, the model shows that legislators are more likely to join a faction if there is a shift in their preferred status quo that is far away from their ideal point.

Moreover, extremist factions, like the Tea Party and the House Freedom Caucus, have garnered disproportionate interest from scholars (e.g., Blum, 2020; Gervais and Morris, 2018). The literature has looked at the effect of activists on Tea Party members (Bailey, Mummolo and Noel, 2012), the corresponding grassroots movement (Skocpol and Williamson, 2012), the effect of economic crises on Tea Party support (Li, 2021), and the joining decisions of Tea Party legislators (Hendry and Sin, 2014). More recent work has investigated the House Freedom Caucus, primarily focusing on its aggressive bargaining tactics (Green, 2019) and their role in obstructing legislation in Congress (Clarke, n.d.). This paper makes a

contribution by drawing theoretical distinctions between legislative behavior in extremist and moderate factions. In particular, this model shows that in the context of a general election, membership in a moderate faction is increasing in reelection benefits, whereas membership in a more extreme faction is only affected by reelection incentives in the context of primaries.

While there does not exist much formal theoretical work of intra-party factions in the U.S. context, there are several models of intra-party politics in the comparative politics literature (e.g., Giannetti and Benoit, 2009; Laver and Shepsle, 1990; Kam, 2009). The work that arguably comes closest to the present paper is Dewan and Squintani (2015), which provides a "defense of factions" and argues that factions may have positive welfare effects, and Invernizzi (2023), which models inter-party and intra-party competition in an electoral context. The model I developed differs in several fundamental ways. First, it examines factions in an accountability (or signaling) framework, and I specifically focus on the U.S. context. Second, importantly, in applying this theory, I consider party welfare in terms of the rewards parties may seek from reelecting their members, instead of the potential benefits from information aggregation. Lastly, in the Appendix, I offer results on the optimal cost of party discipline and endogenous faction platforms.

Beyond the factions literature, this paper speaks to broader lines of existing theoretical research: theories of ideological signaling (Canes-Wrone and Shotts, 2007; Izzo, 2020), parties (Patty et al., 2019), and representation (Hirano and Ting, 2015; Patty and Penn, 2019). This model is most similar to Snyder and Ting (2002, 2003), in that there is uncertainty about the ideology of the incumbent, the voters prefer to minimize the distance between their incumbent and themselves, and incumbents take a roll-call vote which is observed by their voters. The fundamental departures that this paper takes are the notion of intra-party factions as an additional signal to voters, which affects the incumbents' incentives to vote for or against their party, and the results on party welfare. Moreover, the extensions in the Appendix introduce primary elections, explain the existence of non-connected (or discontinuous) roll-call voting coalitions, endogenize the cost of party discipline, derive endogenous

faction platforms, and present results on voter welfare, all of which distinguishes this analysis from earlier work. Finally, this model allows us to analyze differences in majority and minority party voting behavior.

Model

The players in this game consist of a continuum of incumbents $Z := [-1, 1]$ and voters in a continuum of districts $Y := [-1, 1]$. There are two parties in this game: D and R. "Democratic" and "Republican" are used interchangeably with these letters to describe the two major parties. The ideology of each incumbent is denoted by $z \in Z$, and the ideology of the representative (or median) voter in each district is denoted by $y \in Y$. We can think of incumbents/voters to the right of zero as more "conservative" and those to the left as more "liberal."

At the start of this game, a member of Congress with ideal point z in district y chooses to join an intra-party faction ($f \in \{0, 1\}$). For simplicity, I assume that in each party, there is only one faction a member can join, though I consider multiple factions in an extension in the Appendix. A vote takes place, where each incumbent chooses between the party leader's bill ($v = b$) and the status quo ($v = q$). This choice should not be construed as a single roll-call vote but rather as a decision on a broader policy direction or agenda. For instance, the Democratic Study Group (DSG) arose in the mid-twentieth century as a response to the dominance of Southern Democrats and Speaker Rayburn over the legislative agenda on civil rights issues (Bloch Rubin, 2017). The bill and status quo may fall anywhere on the policy space (i.e., $b, q \in [-1, 1]$), though I ignore the trivial case of $b = q$.

Finally, the representative voter in each constituency observes the incumbent's decision to join a faction f and the incumbent's roll-call vote v. The voter chooses to reelect the incumbent or elect a random challenger of the opposite party.

Payoffs

Incumbents

The payoffs for an incumbent with ideal point z in district y can be described as:

$$U(z) = -(z-v)^2 + r(w - m\mathbf{1}(v=q)(1-f)c - (1-m)\mathbf{1}(v=b)(1-f)c) - f(z-x_f)^2$$

This can be broken down into four components: (1) policy utility; (2) winning reelection; (3) reprisal from party leadership; and (4) the cost of joining a faction. First, if an incumbent z votes in favor of the bill ($v = b$), her utility is decreasing in the distance between the bill b and her ideal point z; analogously, if she votes in favor of the status quo ($v = q$), her utility is decreasing in the distance between the status quo q and her ideal point z. Second, incumbents seek to win reelection ($r = 1$) in their constituency and accrue office benefits w.

Third, the cost of party discipline $c(1-f)$ is incurred by a legislator who votes against their party. In the model, I assume this cost is exogenous, but I allow parties to choose the cost of party discipline c in an extension in the Appendix. As implied by the functional form, factions help offset this cost by providing resources (which are black-boxed here). Thus, the cost ultimately applies to non-faction ($f = 0$) majority party ($m = 1$) incumbents who vote for the status quo ($v = q$), as well as non-faction ($f = 0$) minority party ($m = 0$) incumbents who vote in favor of the majority party's bill ($v = b$). Furthermore, the cost of party discipline is only incurred if the incumbent ultimately wins reelection ($r = 1$); one can interpret this assumption as suggesting that party discipline is withheld from incumbents who are likely to lose reelection or who represent electorally competitive seats. I relax this assumption in the Appendix as well.

These payoffs are motivated by previous work that emphasizes the role of factions as a protection against party pressures. Among the resources factions provide, they regularly hold meetings where legislators can establish positions on issues and determine group strat-

egy (Hammond, 2001). Secondly, factions confer electoral benefits by raising funds for their members to increase their chances for winning reelection (e.g., factions have associated political action committees). Finally, in part due to their ability to foster a strong bloc of votes, factions develop connections with party leadership to be able to secure committee posts for their members. I also assume $(2+b+q)|(b-q)| \geq w \geq c \geq max\{b^2-q^2, 0\}$. The first part of this condition ensures that at least some incumbents lose reelection on the equilibrium path, whereas the rest of the condition is merely meant to simplify the analysis. In the Appendix, I discuss how relaxing this condition affects the equilibrium results.

The last term in the incumbents' payoff function is a cost to joining a faction, which is equal to $-f(z-x_f)^2$. This functional form assumes that given the positioning of a faction x_f, the cost of membership is increasing in the distance between the incumbent's ideal point and x_f. While x_f is assumed to be exogenous in the baseline model, I derive endogenous faction platforms in the Appendix, where x_f is the average of the faction members' ideal points. More generally, the quadratic term can be interpreted as the cost for members of undertaking the recruitment process to join a faction. Some of the more institutionally organized factions like the House Freedom Caucus and the Blue Dog Democrats initiate prolonged vetting processes for prospective members. In particular, some require an invitation and sponsor from within the faction to join (Bloch Rubin, 2017; Clarke, 2020; Green, 2019).

Electorate

In each district, there is one representative voter with ideal point y that decides whether to reelect the incumbent or elect the challenger. In this game, voters seek to minimize the distance between their ideal point and their representative's ideal point, but they have some uncertainty about how liberal or conservative their representative is (i.e., the value of the legislator's ideal point z). The median voter's expected payoff is:

$$U(y) = -E_z[(y-z)^2]$$

where y is the ideal point of the voter and z is the ideal point of the incumbent representing the voter. As is often noted, the quadratic functional form can be decomposed into the square of the bias plus the variance. Therefore, we can re-express the payoffs of the voter as

$$U(y) = -(y - \mu(z))^2 - \sigma^2(z)$$

where $\mu(z)$ is the mean and $\sigma^2(z)$ is the variance of the incumbent's ideology z.

Thus, there are two ways that the incumbent can increase the utility of the voter: (1) she can signal that her ideal point is in expectation closer to the ideal point of the voter (i.e., minimize the bias), or (2) she can reduce the interval of possible types (i.e., minimize the variance). I show below that members of Congress can increase their reelection chances by accomplishing one or both of these things.

Beliefs

Voters do not perfectly observe the ideology of their incumbent, though they do observe the incumbent's party affiliation. At the start of the game, in each conservative district (with median $y \geq 0$), Nature randomly draws a Republican incumbent uniformly from the interval $[0, 1]$, and in each liberal district (with median $y < 0$), Nature randomly draws a Democratic incumbent uniformly from the interval $[-1, 0]$. Challengers are drawn in the same way as incumbents. Thus, when voters face a random Republican challenger, for instance, their prior mean for the challenger is $\mu(z) = \frac{1}{2}$, and the prior variance is $\sigma^2(z) = \frac{1}{12}$. Unlike challengers, the voters update their beliefs, using Bayes' Rule, about the incumbents' ideology based on the incumbent's decision to join an intra-party faction (f) and their vote on the proposed agenda (v). The voters' posterior mean and variance of the incumbent's ideology are denoted $\mu(z|f, v)$ and $\sigma^2(z|f, v)$, respectively.[6]

[6]For example, if, in equilibrium, all incumbents $z \in [-1, -\frac{1}{2}]$ in district y affiliate with a faction ($f = 1$) and vote for the status quo ($v = q$) (and all other incumbent types do not), then observing $f = 1$ and $v = q$ yields a posterior mean of $\mu(z|f = 1, v = q) = -\frac{3}{4}$ and posterior variance $\sigma^2(z|f = 1, v = q) = \frac{1}{48}$.

Though this model is written as a one-period game, we can think of the prior beliefs of the voter as interim beliefs, reflecting incumbent behavior that has transpired in previous periods. As a result, we do not necessarily have to think of the incumbent as a first-term politician; we can also interpret her as a third- or fourth-term incumbent who takes actions to further delineate her ideological type. For instance, when the House Freedom Caucus was founded in 2015, some of the original members included Rep. Mark Meadows, a second-term incumbent at the time, and Rep. Raul Labrador, a third-term incumbent at the time.

Game Sequence

The sequencing of the game is:

1. In districts $y \geq 0$, Nature draws a Republican incumbent uniformly from $[0, 1]$, and in districts $y < 0$, Nature draws a Democratic incumbent uniformly from $[-1, 0]$

2. Members of Congress choose to join an intra-party faction $f \in \{0, 1\}$

3. They then vote in favor of the bill ($v = b$) or the status quo ($v = q$)

4. Voters observe f and v and choose to reelect the incumbent or elect a random challenger from the opposite party

Note on Assumptions

Before proceeding to the analysis, it is useful to elaborate on some of the assumptions in the model.

While there is a cost to party discipline from voting against one's own party on the roll-call vote, there is not explicitly a corresponding cost to voting against the rest of one's faction on that roll-call vote. We might think that a cost like this is substantively reasonable since factions, like the House Freedom Caucus, often have institutions in place to punish

defectors (Green, 2019). It is important to note here, however, that there is an implicit cost borne out endogenously in the model because in the equilibria that will be discussed, joining a faction and voting with one's own party is off the equilibrium path. Because there already exists a cost to defecting (equal to w) in the model, adding an additional parameter does not fundamentally change the analysis. It is reasonable that the cost of defection from the faction is a function of reelection benefits, given that the price one pays for defecting on key roll-call votes includes the loss of resources like campaign funds and an electoral brand that are geared towards reelection (Clarke, n.d.; Pearson, 2015).

In addition, this model assumes that faction membership provides resources that *completely* offset the cost of party discipline. This is merely a simplifying assumption. To think about this more formally, denote the resources a faction provides by δ, such that the net cost from voting against one's party is the cost of party discipline minus the resources when one joins a faction (i.e., $c - f\delta$). The model assumes $c = f\delta$, but perhaps a more reasonable specification would state that, since parties have more resources than factions, faction membership under-compensates for the potential threat of retaliation from party leadership (i.e., $f\delta < c$). Under this alternative assumption, the thresholds demarcating which members join a particular faction are merely scaled upwards or downwards. Consequently, relaxing this does not substantively alter the results. This is also true if we were to assume that factions over-compensate for the cost of party discipline (i.e., $f\delta > c$).

Moreover, some have argued that the intended audience for members of party factions just includes donors and party activists (Clarke, 2020), not necessarily the entire constituency of the member of Congress. In the context of this model, one should interpret the signals to the voter represented in the model as being indirect—faction membership leads to additional campaign donations which are used for campaign ads or more endorsements by key interest groups which are then observed by the voter. As such, faction membership operates as a "credential" for legislators observed by donors, and voters update their beliefs based on subsequent media coverage and campaign ads. This interpretation is helpful to keep in mind

when analyzing the main results.

Analysis

This model is solved by starting with period 4, which is the general election stage. In this stage, the voter is comparing her updated beliefs about the incumbent, which consist of the posterior mean $\mu(z|f,v)$ and the posterior variance $\sigma^2(z|f,v)$ to her priors about the challenger from the opposite party. In this section, without loss of generality, I focus on districts $y \leq 0$, which are represented by Democratic incumbents.

One can first note that any incumbent who signals in a "moderate" direction (e.g., $\mu(z|f,v) \geq -\frac{1}{2}$) and makes the voters' posterior more precise ($\sigma^2(z|f,v) \leq \frac{1}{12}$) will win reelection against their challenger. To prove this, one can start by showing that the median voter in district $y = 0$ strictly prefers to reelect the incumbent. The expected mean of the incumbent is closer to zero than the expected ideological mean of the challenger (i.e., $\frac{1}{2}$). Thus, the bias is smaller in this case, as is the variance. If the mean of the incumbent's ideology is closer to $y = 0$ than $\frac{1}{2}$, then it is closer to any $y < 0$ as well. This means that since the median voter in district $y = 0$ prefers to reelect the incumbent, all median voters in districts $y < 0$ prefer to reelect the incumbent. As a result, any incumbent whose expected type is weakly more moderate than the prior gets reelected in the general election. This is described formally in the first part of Lemma 1.

Lemma 1: If $\sigma^2(z|f,v) \leq \frac{1}{12}$, the general election voter's reelection decision in districts $y \leq 0$ can be described as follows:

- If $\mu(z|f,v) \geq -\frac{1}{2}$, then the voter always reelects (i.e., $r = 1 \ \forall y \leq 0$).

- If $\mu(z|f,v) < -\frac{1}{2}$, then the voter reelects if

$$y \leq \frac{1 - 3(\mu(z|f,v))^2 - 3\sigma^2(z|f,v)}{3(1 - 2\mu(z|f,v))}$$

Proof of Lemma 1: All proofs can be found in the Appendix

The interesting tradeoff, however, exists when the incumbent signals as "extreme" (i.e., $\mu(z|f,v) < -\frac{1}{2}$). In this case, the expected mean of the incumbent's type is in some cases further away from the median voter's ideal point than the challenger's. Thus, the voter only reelects if she is sufficiently ideologically extreme in this case (i.e., when the reduction in variance sufficiently outweighs the loss, if any, in policy utility). This condition is given in the second part of Lemma 1. This result can be interpreted as saying that an incumbent can only survive the general election while signaling as "extreme" if her constituents are similarly ideologically inclined.

Equilibrium Faction Membership and Roll-Call Voting

In this section, incumbent behavior prior to the election stage is described. A common characteristic of signaling games, such as this one, is the presence of multiple equilibria. Since one can always justify an equilibrium in which no legislators join a faction, this analysis only seeks to outline conditions under which we observe factions in equilibrium. The following section details *majority* party factions, whereas discussion of minority party factions is reserved for the Appendix. In this analysis, without loss of generality, the majority party is assumed to be Democratic.

Rightward Shift in the Status Quo ($q < b$)

When $q < b$, there exists an equilibrium in which the most liberal incumbents in the most ideologically extreme districts join a faction and vote against their party's proposed bill.

This equilibrium requires the platform of the majority party faction (x_f) to be sufficiently liberal so that the cost from joining a faction cannot be so demanding that even ideologically extreme members would rather just vote with their party. Reelection benefits (w) have to be large enough so that some members would not have an incentive to deviate later in the roll-call voting stage and vote for the status quo instead of the bill. The equilibrium is shown visually in Figure 1 and described formally in Proposition 1. The full details of the equilibrium are given in the Appendix.

Proposition 1: When there is a rightward shift in the status quo ($q < b$), the faction platform is sufficiently liberal ($x_f \leq \underline{x}_f$), and reelection benefits are sufficiently high ($w \geq \tilde{w}$):

- Incumbents $z \in [-1, z_1^{ext}]$ in districts $y \in [-1, y_1^{ext}]$ join an intra-party faction and vote for the status quo, whereas incumbents $z \in (z_1^{ext}, 0]$ vote for the party's bill and do not join a faction. Incumbents in all other districts abstain from faction membership.

In constituencies that are ideologically extreme (i.e., $y \leq y_1^{ext}$), the incentives for legislators to join a faction are a function of the status quo (q), the bill (b), and the faction platform (x_f). Incumbents z are incentivized to join a faction if they meet the following inequality:

$$\overbrace{-(z - x_f)^2}^{\text{Cost of faction membership}} + \overbrace{-(z - q)^2}^{\text{Policy utility from status quo}} \geq \overbrace{-(z - b)^2}^{\text{Policy utility from bill}} \quad (1)$$

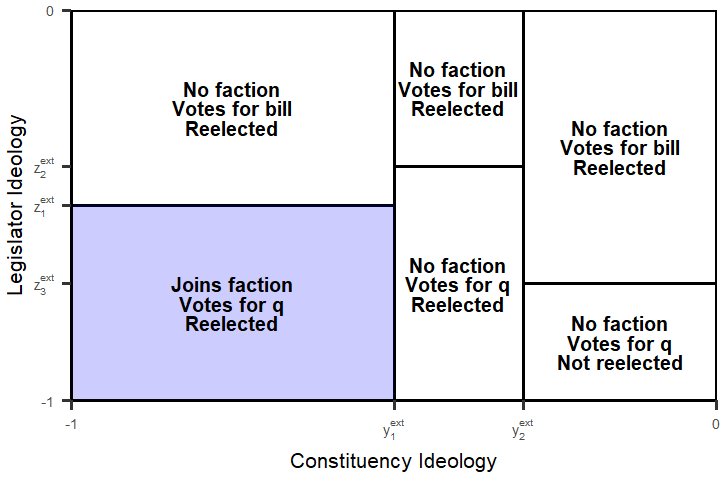
As one can see above in (1), incumbents weigh the utility from voting for the status quo and the cost of faction membership (LHS) against the utility from the majority party's agenda (RHS). Democrats who prefer to maintain the status quo (versus the proposed bill) and are willing to expend the cost of membership (i.e., the squared distance between the faction platform and the incumbent's ideal point), i.e., $z \in [-1, z_1^{ext}]$, will join an intra-party faction.

To justify this equilibrium, one has to further show that incumbents do not deviate in the subsequent roll-call voting stage. This requires that those who originally abstained from

faction membership do not later on vote for the status quo. As noted earlier, this is satisfied when reelection benefits (w) are sufficiently large, since incumbents lose w when choosing off-equilibrium path actions (i.e., not joining a faction but voting for the status quo). We also require that faction members do not deviate and vote later on for the majority party's agenda; this is always true since condition (1) is more restrictive than the corresponding condition in the roll-call voting stage.

As Lemma 1 implies, an equilibrium in which some incumbents join a faction and get reelected only exists in a certain interval of districts $[-1, y_1^{ext}]$, since incumbents who signal as "extreme" can only win reelection in ideologically extreme districts. Using Lemma 1, one can derive the threshold y_1^{ext}, which denotes the least liberal district that is willing to reelect the members that signal as "extreme" in equilibrium by joining a faction. Put differently, any district to the right of y_1^{ext} is too moderate to reelect these "extreme" members, because the average Republican challenger is closer to the district's ideal point than the average (extreme) faction member.

Figure 1: Extreme Party Factions ($q = -1$, $b = 0$, $c \leq \hat{c}$)



One can now analyze the rest of the parameter space shown in Figure 1. In districts $[y_1^{ext}, y_2^{ext}]$, where faction membership is off the equilibrium path, liberal legislators vote for the status quo and incur the cost of party discipline c whereas others vote for the bill. This

region only exists when c is sufficiently small, or when $z_1^{ext} \leq z_2^{ext}$. Otherwise, those who vote for the status quo are too extreme, on average, to win reelection in these districts. Like with y_1^{ext}, the threshold y_2^{ext} denotes the least liberal district that is willing to reelect the members who vote for the status quo.

Finally, in the most moderate districts (i.e., $y \in [y_2^{ext}, 0]$), incumbents consider the level of reelection benefits w in their voting decisions; this is because some incumbents lose reelection if they do not cater to the middle and vote for the bill. More generally, in this interval of districts, incumbents signal as "moderate" for at least one of two reasons: (1) policy gains; and (2) reputational concerns. The first reason is relatively straightforward; these incumbents favor the bill over the status quo and want to vote in accordance with their preferences. However, incumbents also vote for the bill because they want to appear sufficiently aligned with their moderate constituencies. If they vote for the more extreme status quo, they signal to their districts that they belong to a more liberal interval of legislators and consequently lose reelection (as Lemma 1 suggests). This is why, in equilibrium, there may be some legislators who actually prefer the status quo, ideologically speaking, over the bill, but still vote for their party's agenda anyway. In this sense, reputational concerns motivate legislators to sometimes abandon their ideological inclinations.

Leftward Shift in the Status Quo ($q > b$)

Now we consider the case of $q > b$. In equilibrium, the most conservative Democrats in the most moderate districts join a faction, whereas more liberal incumbents in these districts abstain from faction membership. When there is a leftward shift in the status quo, this equilibrium requires a sufficiently conservative faction platform (x_f) and a sufficiently high level of reelection benefits (w). These conditions are required to ensure that at least some Democratic incumbents prefer to join a faction. If reelection benefits are too small, then no incumbents prefer to join a faction. If the faction platform is too liberal, then faction membership is too expensive for some moderate Democrats. This equilibrium is shown

visually in Figure 2 and described in Proposition 2. The full details are in the Appendix.

Proposition 2: When there is a leftward shift in the status quo ($q > b$), the faction platform is sufficiently conservative ($x_f \geq \overline{x}_f$), and reelection benefits are high enough ($w \geq q^2 - b^2 + (x_f)^2$):

- Incumbents $z \in [z_3^{mod}, 0]$ in districts $y \in [y_2^{mod}, 0]$ join a faction and vote for the status quo, whereas incumbents $z \in [-1, z_3^{mod}]$ abstain from faction membership and vote for the bill. Incumbents in all other districts abstain from faction membership.

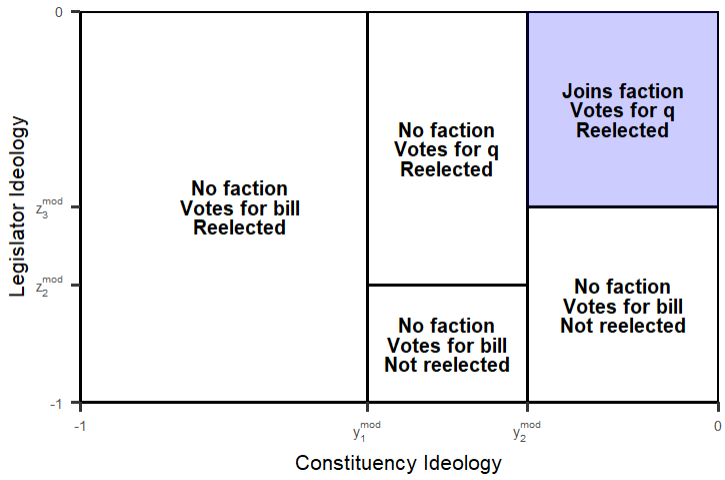
In districts $[y_2^{mod}, 0]$, as mentioned before, moderate Democrats join a faction and win reelection, whereas those who are sufficiently liberal forego faction membership. Incumbents z are incentivized to join a faction when the following inequality holds:

$$\overbrace{w}^{\text{Reelection benefits}} + \overbrace{-(z - x_f)^2}^{\text{Cost of faction membership}} + \overbrace{-(z - q)^2}^{\text{Policy utility from status quo}} \geq \overbrace{-(z - b)^2}^{\text{Policy utility from bill}} \quad (2)$$

On the left hand side of equation (2), incumbents derive reelection benefits from faction membership, the cost of faction membership, and a policy loss from voting for the status quo; on the right hand side, when abstaining from faction membership, incumbents simply incur the policy loss from voting for the bill. Interestingly, when considering moderate factions, the size of the interval of legislators who join a faction is increasing in reelection benefits, which is not true for extreme factions. The reasoning is that faction membership in these electorally competitive districts is part of a *necessary* condition for reelection (whereas in the case of extreme factions in the previous section, non-faction members can also win reelection on the equilibrium path). As such, when the benefits from reelection increase, legislators are more inclined to expend the cost of faction membership. Note here that $[y_2^{mod}, 0]$ is the largest possible interval of districts in which this equilibrium can be justified;

y_2^{mod} denotes the most liberal district that is not willing to reelect the incumbents who vote for the proposed agenda.

Figure 2: Moderate Party Factions ($b = -1, q = 0, c \leq \tilde{c}$)



Considering the rest of the parameter space in Figure 2, in districts $[y_1^{mod}, y_2^{mod}]$, liberal incumbents vote for the party's agenda, and more moderate incumbents vote for the status quo; all incumbents here abstain from faction membership, but only the latter group wins reelection. Note that this equilibrium only exists when the cost of party discipline is sufficiently low or reelection benefits are sufficiently high. The key tradeoff motivating incumbents' decision-making here is that those who vote for the proposed agenda lose reelection in these districts, but a vote for the status quo incurs the cost of party discipline c. Indeed, as reelection benefits increase, more incumbents vote for the status quo, and as the cost of party discipline increases, more incumbents vote for the party's agenda. Lastly, in ideologically extreme districts $[-1, y_1^{mod}]$, all incumbents vote for the bill and win reelection.

Welfare Analysis

One can now explore the welfare implications of factions. To perform this welfare analysis, comparisons are made between the equilibria described above and what one can refer to as

the "no factions" equilibria, in which there are no faction members but legislators still cast votes. These equilibria are described in Lemma 2 and shown visually in Figure 3.

First consider $q < b$. In the most extreme districts, legislators' votes are a function of their policy preferences and the cost of party discipline. Consistent with the result in Lemma 1, a strategy profile in which all Democratic incumbents win reelection on the equilibrium path can only be justified in sufficiently liberal districts; in the most moderate districts, legislators who send the "extreme" signal lose reelection. As such, in these latter, more moderate districts, reelection incentives play a role in the decisions for legislators to cast votes. Legislators still care about policy, but they may be incentivized to vote for the more moderate policy because of reputational concerns.

Now consider $q > b$. In the most moderate districts, legislators face a tradeoff: either vote for the bill and risk appearing too extreme to one's constituents and lose reelection or vote for the status quo and incur the cost of party discipline (while winning reelection). Larger reelection benefits encourage legislators to vote for the bill whereas more stringent party discipline induces support for the status quo. In the most ideologically extreme constituencies, all legislators vote for the bill, because the cost of party discipline is sufficiently high (and voting for the bill does not lead to losing reelection in this region).

Lemma 2 (No Factions):

- If $q < b$, incumbents $z \in [-1, z_1^{NF,ext}]$ in districts $[-1, y_1^{NF,ext}]$ and incumbents $z \in [-1, z_2^{NF,ext}]$ in districts $[y_1^{NF,ext}, 0]$ vote for q (and all others vote for b).
- If $q > b$ and $w - c \geq q^2 - b^2$ or $b + q \leq 0$, incumbents $z \in [z_2^{NF,mod}, 0]$ in districts $[y_{mod}^{NF}, 0]$ vote for q, and all others vote for b.

This is a reasonable equilibrium to consider as a benchmark comparison, because no incumbents are a member of a faction, and as such, factions do not play any role whatsoever

in determining how legislators vote. To simplify the nomenclature in this section, I refer to the equilibrium in Lemma 2 as the NF equilibrium, the result in Proposition 1 as the "extreme faction" equilibrium, and the result in Proposition 2 as the "moderate faction" equilibrium.

Party Welfare

In this section, I assume parties care about: (1) encouraging legislators to vote in favor of the party's agenda; and (2) reelecting their own members so that they can attain majority status. In both of these components, as will be noted below, I consider separately whether the party exclusively cares about the *absolute* proportion of legislators voting for the bill and getting reelected or just about the threshold cases (e.g., 50% + 1 of legislators needed to pass a bill or 50% + 1 of legislators needed to keep the party in the majority). Results on voter welfare are described in the Appendix.

The first component is informed by literature that suggests that party leaders want to avoid getting "rolled" on the floor (Cox and McCubbins, 2005). In each legislative session, party leaders typically have a legislative agenda that they seek to pass. As such, this component is designed to reflect desires of party leadership to secure policy achievements by the end of the session. The latter component is motivated by work that says that party leaders care about retaining majority party status and reelecting their party members (Green and Harris, 2019). In a time of insecure majorities (Lee, 2016), it is crucial for leaders to enable their members to signal their ideological alignment to their constituencies, so that the party holds onto as many seats as possible in the next election. As Pearson (2015) notes, "even frequent party defectors in tight races receive assistance from party leaders" (page 146).

Given this conceptualization, we can define the parties' expected welfare in two ways:

$$W_{P1} = \int \int (\mathbf{1}(v = b) + \alpha mr) dy dz$$

$$W_{P2} = Pr(\text{Bill Passes}) + \alpha Pr(\text{Majority of Seats Held by Party})$$

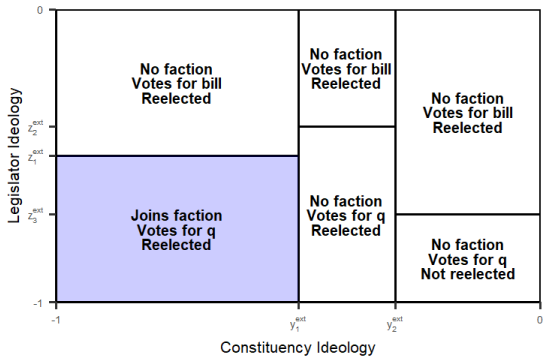
In the first specification, v is the legislator's vote, m is legislator z's majority-party status, r is whether legislator z gets reelected in district y, and α represents the weight parties place on reelecting their members relative to passing the bill. The first term in the first function represents the region in which members vote for the proposed bill, and the second term represents the region in which majority party members get reelected. One should note that v, m, and r are functions of y and z, such that the welfare function integrates over the behavioral/electoral outcomes across districts and legislators.

One could argue here, instead, that parties might care more about the threshold cases, in which they lose the majority of seats or lose the majority of votes for the bill. In the Appendix, I use the second welfare function, W_{P2}, to incorporate these thresholds and consider a discrete version of the model where some legislators can be pivotal. The results are substantively similar to those presented below, which focus on the first welfare function.

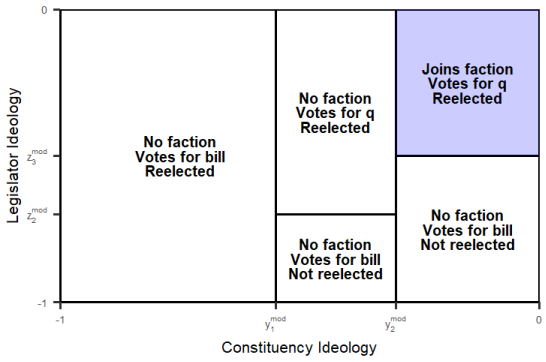
When comparing the NF equilibrium with the one in Proposition 1, which are both shown in Figure 3 within the context of the same parameter space, Proposition 3 indicates when factions yield greater welfare for parties. If the weight that parties place on reelecting their members is sufficiently high ($\alpha \geq \hat{\alpha}$) and factions are less restrictive in their recruitment ($z_1^{ext} \leq z_1^{NF,ext}$ or equivalently, $x_f \geq \hat{x}_f$), then factions have a net positive effect on party welfare. There is a key tradeoff in this case. Factions that are less restrictive in their recruitment enable a larger interval of incumbents to vote against their party's agenda. On the other hand, the interval of districts in which some incumbents get thrown out of office is smaller. Thus, if parties care more about the latter effect than they do about the former (i.e., $\alpha \geq \hat{\alpha}$), then ideologically inclusive factions can be a welfare improvement for parties. If factions are more restrictive in their recruitment ($z_1^{ext} \geq z_1^{NF,ext}$ or equivalently, $x_f \leq \hat{x}_f$), then no tradeoff exists here: fewer incumbents vote against the party's agenda, and party welfare increases.

Figure 3: Comparison Between Factions and "No Factions" Equilibria

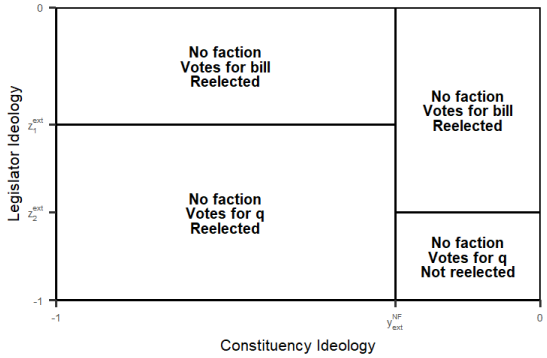
(a) Proposition 1: Extreme Factions ($q < b$)



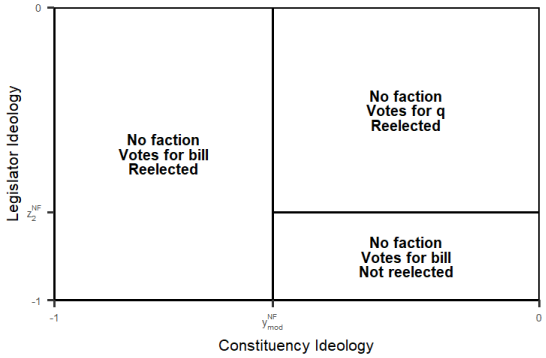
(b) Proposition 2: Moderate Factions ($q \geq b$)



(c) Lemma 2: No Factions ($q < b$)



(d) Lemma 2: No Factions ($q \geq b$)



Proposition 3: The extremist factions equilibrium accrues greater party welfare if and only if $z_1^{ext} \leq z_1^{NF,ext}$ (or equivalently, $x_f \geq \hat{x}_f$) or $z_1^{ext} > z_1^{NF,ext}$ and $\alpha \geq \hat{\alpha}$. The moderate factions equilibrium accrues greater party welfare if and only if $z_3^{mod} \leq z_2^{NF,mod}$ and $\alpha \geq \hat{\alpha}$.

Now, we can consider the case of moderate factions. If a larger interval of moderate incumbents vote against the party ($z_3^{mod} \leq z_2^{NF,mod}$ or equivalently, $x_f \geq \tilde{x}_f$), parties once again face a tradeoff. On the one hand, parties face greater difficulty passing their legislative agenda. On the other hand, however, there is greater pooling among incumbent types, leading to higher reelection rates in these districts. In other words, more party members are winning reelection because they can take advantage of the signal associated with faction membership in electorally competitive districts. Thus, parties can benefit when moderate

factions are more ideologically inclusive, particularly when there is substantial weight placed on reelecting their members ($\alpha \geq \hat{\alpha}$). This is described in the second part of Proposition 3.

Case Studies

I present two case studies examining the role of intra-party factions on key roll-call votes. One should note that these cases do not comprise a test of the model. Rather, they serve to elucidate certain components of the model. This exercise is important because the model is designed to reflect real-world phenomena in the U.S. Congress.

The first case illustrates an example of the equilibrium in Proposition 1 (i.e., an extreme party faction—the House Freedom Caucus), whereas the second case is an example of the equilibrium in Proposition 2 (i.e., a moderate party faction—the Blue Dog Coalition).

Funding Measure for the Department of Homeland Security

The House Freedom Caucus (HFC) was founded in early 2015 by several conservative representatives, including Rep. Ron DeSantis (R-FL) and Rep. Mark Meadows (R-NC), in order to shift policy to the right in the then-Republican-controlled House. Much like the equilibrium faction described in Proposition 1 (if one flipped the axes to look at the Republican side), this faction consisted of very conservative incumbents who represented heavily Republican districts. In the context of the model, we can think of the HFC's faction platform as equal to $x_f = 1$, meaning that recruitment is largely restricted to very conservative legislators. Indeed, the House Freedom Caucus put in place an invitation-only policy and created membership caps (Bloch Rubin, 2021). In addition, they required members to pay dues (Green, 2019).

In this section, I focus on a specific roll-call vote—the annual funding measure for the Department of Homeland Security (DHS)—where members of the House Freedom Caucus defected from their party. Consistent with the condition described in Proposition 1, this was

a case in which conservatives largely sided with an extreme conservative status quo. In late February 2015, the GOP-led Senate had passed a clean full-year funding bill for the DHS, and time was running out to extend funding. Indeed, failure to pass this bill in the House would have led to a shutdown of the department. More generally, this was a turbulent time for House Republicans and intra-party factions. At the start of FY2014, the HFC antecedent (Tea Party Caucus) engineered the longest shutdown in American history. Towards the end of negotiations between the House and Senate on a bipartisan budget package the following year, President Obama used executive action to unilaterally reform DACA/DAPA. The DHS funding bill then became the focal point on whether to attach a rider prohibiting President Obama's executive action.

Much like in the model, it is clear that incumbents' votes on this measure were at least partially motivated by electoral pressures. The bill ultimately passed the House with largely Democratic support (257-167), with only 75 Republicans voting for the bill. Rep. Jim Jordan at the time said in an interview, "we told the voters [the vote on the DHS funding measure] was the defining moment. We said we would make sure that no funds could be used in any way to carry out [Obama's] unlawful action."[7] Most Republicans, like Rep. Sessions and Rep. Jenkins, who voted against the bill "sa[id] they couldn't support a measure that didn't explicitly block the immigration actions" by President Obama.[8] Conservatives thought President Obama's action was "unconstitutional" because it was done via executive fiat and not through the legislative process in Congress. In response to this opposition to the measure, "the nonprofit American Action Network [which is affiliated with the House leadership] [aired] an ad in three states — Kansas, Oklahoma and Ohio — urging Reps. Tim Huelskamp (R-Kan.), Jim Bridenstine (R-Okla.) and Jim Jordan (R-Ohio) to back funding for the [DHS]."[9] The group had also run "radio ads on nationally syndicated shows and digital

[7] *The Atlantic*, "Boehner Has a Plan to Avoid a DHS Shutdown—But It Might Not Pass," February 26, 2015.

[8] *Roll Call*, "House Agrees to Fund Homeland Security Department," March 3, 2015.

[9] *Politico*, "GOP group targets House conservatives on DHS fight," March 2, 2015.

ads in dozens of other districts represented by House conservatives." Indeed, if one looks into the final vote, there is much evidence that conservative Republicans representing conservative districts largely voted against the legislation. Strikingly, none of the HFC members voted in favor. A more formal breakdown of the roll-call vote by constituency ideology, legislator ideology, and HFC membership is provided in Table C.1 in the Appendix.

Finally, many representatives, particularly those in the House Freedom Caucus, had faced party discipline from party leadership for some of their votes in 2015, which came in many forms. *The New Yorker* reports that "[Rep.] Mulvaney was kicked off a subcommittee that he chaired." In addition, "[Rep.] Duncan, the chairman of the Subcommittee on the Western Hemisphere, which oversees American policy toward Latin America, says that he wasn't allowed to go on international congressional trips, a normal perk for most members."[10,11,12] Consistent with the assumption the model makes, Clarke (n.d.) shows that the HFC provided resources to help members, to ease the consequences of punishments that parties hand down to defectors. For instance, HFC staff had taken advantage of a procedural rule to restore Rep. Mark Meadows's position on the Oversight Committee, which had earlier been rescinded.

Thus, the case described above elucidates several components of the theory. First, more conservative members, particularly those in the House Freedom Caucus, voted against their party in favor of an extreme conservative status quo. Second, they did so for ideological reasons—they wanted to restrict President Obama's executive order on immigration—but importantly, they were also responding to electoral pressures from their constituency. Finally, the HFC served as an avenue for members to retain access to resources, in the event that their votes against party leadership would be met with reprisal.

[10] *The New Yorker*, "A House Divided," December 6, 2015.

[11] *New York Post*, "Freedom Caucus member reveals GOP's 'vindictive retaliation'," April 8, 2017.

[12] In an op-ed for the *New York Post*, Rep. Ken Buck notes that "three members of the Republican Whip team, who voted against their party, were removed from their positions — Trent Franks of Arizona, Cynthia Lummis of Wyoming, and Steve Pearce of New Mexico."

Blue Dog Democrats: 111th Congress

The Blue Dog Coalition first formed after the 1994 elections, when Republicans swept into power in the U.S. House in a historic landslide victory (Bloch Rubin, 2017). The Blue Dog Democrats are a small faction that brands themselves as fiscally conservative. In the context of the model, we can think of this group's faction platform as residing closer to 0, in the sense that they are very ideologically exclusive, and they implement strict ideological tests of their members, e.g., they are invitation-only and require a sponsor from within their faction (Bloch Rubin, 2021; Clarke, 2020). In the 2009-2010 session, they had over 40 members. As the equilibrium depicted in Figure 2 suggests, these members largely comprised conservative Democrats in moderate districts.

This section examines a vote on the American Clean Energy and Security Act ("Waxman-Markey") in 2009, in which members of the Blue Dog Democrats defected from their party. At this time, Democrats controlled both chambers in Congress and the presidency. Taking advantage of their status as a unified government, President Obama and Democratic leaders in Congress had embarked on a comprehensive legislative agenda that famously included healthcare reform and a large stimulus package aimed at restoring economic growth after the Great Recession. As part of this agenda, Speaker Pelosi in the House tried to pass environmental legislation, in the form of Waxman-Markey.

Note that much like the condition invoked in Proposition 2, this bill marked a leftward shift in the status quo. Waxman-Markey sought to place a cap on the total number of greenhouse gases emitted in the United States, under which companies can then buy and sell permits to emit CO_2, or carbon dioxide, to each other in a way that operates as a market for greenhouse gases. Democrats largely argued in favor of the bill, promulgating its positive impact on the environment and efforts to "improve national security, create jobs... and reestablish the United States as a world leader."[13] The Republicans largely opposed the

[13] *Politico*, "House passes climate change bill," June 25, 2009.

bill, arguing that it would raise electricity prices and impose regulations on businesses. In rather stark terms, Speaker Boehner publicly stated that Waxman-Markey was the "biggest job-killing bill that's ever been on the floor of the House of Representatives."

In part because of this Republican opposition, some members of Congress, particularly conservative Democrats in moderate districts like the Blue Dogs, were apprehensive about how their votes on this bill would affect their reelection prospects. Rep. John Salazar (D-CO), for instance, stated "I'm in a tough spot. I really am." He was afraid that the "legislation could raise energy costs and hurt the coal industry in his low-income, rural district." Adding to this pressure, many special interests had indicated that this vote would factor into their endorsements. Specifically, "the U.S. Chamber of Commerce reiterated its opposition [to the cap-and-trade bill] in a letter to lawmakers...that warned it would consider the votes in its annual scorecard. The group [had] donated to several lawmakers who... are fence-sitters on the climate bill."[14]

One result in the model is that Democrats in moderate districts are more likely to lose reelection if they vote with their party, if there is a leftward shift in the status quo. While it is hard to know the counterfactual outcome had moderate Democrats voted differently on Waxman-Markey, there is some anecdotal evidence that support for the bill may have yielded some negative electoral consequences in the 2010 elections. For example, Rep. Rick Boucher of western Virginia who supported the cap-and-trade bill, "lost in a shocker to Republican Morgan Griffiths in a campaign dominated by coal." Speaking in hindsight, the former campaign spokesman to Boucher stated "I don't think there's any question about it, cap-and-trade was the issue in the campaign" and "if Rick had voted no, he wouldn't have had a serious contest [in the 2010 elections]."[15]

As evidence of further electoral punishment, many interest groups had funded the challengers of those who voted in favor of the cap-and-trade bill. After witnessing the votes on

[14] *E & E Daily*, "House Dems prepare to gamble on cap-and-trade vote," June 26, 2009.

[15] *Time*, "Politics: How Much Did Cap-and-Trade Hurt the Democrats? Not As Much As You Think," November 3, 2010.

cap-and-trade, in addition to those on the Affordable Care Act and Dodd-Frank, "many businesses and trade groups discovered that their campaign investments [to moderate Democrats] had not always paid off."[16] Consequently, "following these votes, some PACs cut their losses or even sent checks to Republican opponents." As suggested earlier, many of the incumbents who voted against the bill were conservative Democrats in rural parts of the country, consistent with the prediction of the model. A formal breakdown of this vote, by constituency and legislator ideology, is provided in Table C.2 in the Appendix.

This case highlights several components of the theory. First, moderate Democrats coalesced together to vote against a more liberal bill in favor of a more conservative status quo. Second, they did so because of reelection pressures and concerns from their heavily rural districts. Finally, anecdotal evidence suggests that those in moderate districts that did vote for the bill received some electoral consequences; interest groups withheld funds, challengers were propped up during the campaign, and some incumbents ultimately lost reelection in their districts.

Conclusion

This model reveals many intuitions about the strategic decisions made by U.S. lawmakers to join factions and defect from their party. In equilibrium, members of Congress are most incentivized to join a faction when they are relatively opposed to their party's agenda. Parties propose policies that shift the status quo, and faction membership in the model alleviates the cost of voting against one's own party. As a result, when incumbents face a policy agenda that is more moderate relative to the status quo, ideologically extreme members are inclined to join a faction (and vice versa).

This theoretical framework relied on a few key ingredients. The first is that institutional organizations like factions provide resources to help offset the cost of party discipline that

[16] *Public Integrity*, "PACs flocked to conservative Blue Dog Democrats, then fled after crucial votes," December 7, 2010.

party leaders sometimes hand down to dissident members, or those that fail to toe the party line on key roll-call votes. The second is that there are informational asymmetries between voters and incumbents, such that voters are uncertain about the incumbent's ideological preferences. To remedy some of these informational gaps, incumbents signal their preferences to their districts through roll-call voting and faction membership in order to ward off general election challengers. The model assumes that voters begin with some prior information about the incumbent given their party affiliation, but intra-party labels and roll-call votes provide further delineation.

Ultimately, this theory sheds light on the question of whether or not factions lead to greater welfare gains for parties. While scholars suggest that hardline factions are perceived in a negative light, due to their propensity for internecine conflict (e.g., Boucek, 2009; Dewan and Squintani, 2015), the theory presented here shows that ideologically exclusive factions can increase party welfare because they communicate the ideological type of the incumbent to constituents, improving the incumbents' reelection prospects, thereby helping the party retain majority status. In this sense, parties might be better off preserving factions even if it means more difficulty, in the short term, in passing their legislative agenda. As such, groups like the House Freedom Caucus and the Blue Dog Democrats, as depicted in the case studies, can on balance be helpful to parties, to the extent that they provide an avenue for legislators to signal their ideological alignment to the voters.

The welfare results might inform how we think about the implications of new ideological factions that have been forming in Congress in recent years. For example, the controversial "America First" caucus, reportedly led by Rep. Marjorie Taylor-Greene (R-GA) and Rep. Paul A. Gosar (R-AZ) in 2021, touted a "nationalist approach to issues such as immigration, trade and foreign policy."[17] Given some of the incendiary positions espoused by some of its members, the caucus received backlash from Republican leaders, like Minority Leader Kevin

[17] *The Washington Post*, "Rep. Greene tries to distance herself from 'America First Caucus' document denounced as racist," April 17, 2021.

McCarthy. The theoretical framework laid out in this paper suggests, however, that a small faction like this presents an opportunity for members to signal their support for a nationalist agenda and for constituents to better evaluate whether their incumbent is congruent with their preferences. On the Democratic side, there has been much discussion about forming an ideologically exclusive hardline group much like the House Freedom Caucus. Again, while groups like the Squad have rankled party leadership in the past, these developments may have positive implications in the sense that these groups will give voters more information about their incumbent.

Moreover, this paper explores some extensions to the model which are presented in the Appendix, such as introducing primary elections, deriving the optimal cost of party discipline, and endogenizing the faction platform. In these extensions, the theory shows that primaries induce greater membership in ideologically extreme factions, due to electoral pressures to appear more extreme in less moderate districts. In addition, by raising the cost of party discipline, the theory demonstrates that parties may face a tradeoff: greater numbers of legislators vote in favor of the policy agenda but these legislators may lose reelection in their districts by potentially signaling as extreme. Additional extensions explain the existence of non-connected roll-call voting coalitions and multiple factions in the majority party.

Finally, one can consider future avenues for additional theory-building in the area of intra-party factions. As mentioned before, while there is abundant theoretical work on intra-party politics in the comparative politics literature and in the context of countries outside the United States, there has been little development of formal theories on factions in the U.S. context. Given the framework in this paper, one could answer questions about the optimal level of recruitment on the part of factions and the extent to which the creation of organized factions can lead to policy gains. In particular, it is unclear how small or large factions should be in order to achieve their desired policy aims. This question has practical relevance given recent discussions about whether there should be an ideologically exclusive *progressive* faction on the left, as a counterpart to the House Freedom Caucus on the right.

References

Aldrich, J.H. 1995. *Why Parties?: The Origin and Transformation of Political Parties in America*. American Politics and Political Economy Series University of Chicago Press.

Bailey, Michael A., Jonathan Mummolo and Hans Noel. 2012. "Tea Party Influence: A Story of Activists and Elites." *American Politics Research* 40(5):769–804.

Bawn, Kathleen, Martin Cohen, David Karol, Seth Masket, Hans Noel and John Zaller. 2012. "A Theory of Political Parties: Groups, Policy Demands and Nominations in American Politics." *Perspectives on Politics* 10(3):571–597.

Bloch Rubin, Ruth. 2013. "Organizing for Insurgency: Intraparty Organization and the Development of the House Insurgency, 1908-1910." *Studies in American Political Development* 27(2):86–110.

Bloch Rubin, Ruth. 2017. *Building the Bloc: Intraparty Organization in the US Congress*. Cambridge University Press.

Bloch Rubin, Ruth. 2021. "Organizing at the Extreme: Hardline Strategy and Institutional Design." *Working Paper* .

Blum, Rachel M. 2020. *How the Tea Party Captured the GOP*. The University of Chicago Press.

Boucek, Françoise. 2009. "Rethinking Factionalism: Typologies, Intra-Party Dynamics and Three Faces of Factionalism." *Party Politics* 15(4):455–485.

Canes-Wrone, Brandice and Kenneth W. Shotts. 2007. "When Do Elections Encourage Ideological Rigidity?" *American Political Science Review* 101(2):273–288.

Clarke, Andrew. n.d. "The House Freedom Caucus: Extreme Faction Influence in the U.S. Congress." *Working Paper* .

URL: *https://www.democratic-anxieties.eu/wordpress/wp-content/uploads/2017/10/Clarke_Berlin2017.pdf*

Clarke, Andrew J. 2020. "Party Sub-Brands and American Party Factions." *American Journal of Political Science* .

Cohen, Marty, David Karol, Hans Noel and John Zaller. 2008. "The Party Decides: Presidential Nominations Before and After Reform.".

Cox, Gary W. and Mathew D. McCubbins. 2005. *Setting the Agenda: Responsible Party Government in the U.S. House of Representatives*. Cambridge University Press.

Cox, G.W. and M.D. McCubbins. 1993. *Legislative Leviathan: Party Government in the House*. California Series on Social Choice and Political Economy University of California Press.

Dewan, Torun and Francesco Squintani. 2015. "In Defense of Factions." *American Journal of Political Science* 60(4):860–881.

Gervais, Bryan and Irwin L. Morris. 2018. *Reactionary republicanism : how the Tea Party in the House paved the way for Trump's victory / Bryan T. Gervais and Irwin L. Morris.* New York, NY: Oxford University Press.

Giannetti, Daniela and Kenneth Benoit. 2009. *Intra-Party Politics and Coalition Governments*. Routledge/ECPR Studies in European Political Science.

Gilmour, John B. 1995. *Strategic Disagreement: Stalemate in American Politics*. University of Pittsburgh Press.

Green, Matthew. 2019. *Legislative Hardball: The House Freedom Caucus and the Power of Threat-Making in Congress*. Elements in American Politics Cambridge University Press.

Green, Matthew N. and Douglas B. Harris. 2019. *Choosing The Leader*. Yale University Press.

Hammond, S.W. 2001. *Congressional Caucuses in National Policymaking*. Johns Hopkins University Press.

Hendry, David J. and Gisela Sin. 2014. "Joining the Tea Party Caucus: A Survival Strategy." *Working Paper* .

Hirano, Shigeo and Michael M. Ting. 2015. "Direct and Indirect Representation." *British Journal of Political Science* 45(3):609–634.

Invernizzi, Giovanna M. 2023. "Antagonistic Cooperation: Factional Competition in the Shadow of Elections." *American Journal of Political Science* 67(2):426–439.

Izzo, Federica. 2020. "With Friends Like These, Who Needs Enemies?" *Working Paper* .
URL: *https://www.federicaizzo.com/pdf/WFLT10K.pdf*

Kam, Christopher J. 2009. *Party Discipline and Parliamentary Politics*. Cambridge University Press.

Laver, Michael and Kenneth A. Shepsle. 1990. "Coalitions and Cabinet Government." *The American Political Science Review* 84(3):873–890.

Lee, F.E. 2009. *Beyond Ideology: Politics, Principles, and Partisanship in the U. S. Senate*. University of Chicago Press.

Lee, F.E. 2016. *Insecure Majorities: Congress and the Perpetual Campaign*. University of Chicago Press.

Li, Zhao. 2021. "How Economic Crises Shape Political Extremism in the United States: the Role of Campaign Donors." *Working Paper* .

Madison, James. 1788. *The Federalist No. 10*. The Federalist Papers.

McCarty, Nolan and Eric Schickler. 2018. "On the Theory of Parties." *Annual Review of Political Science* 21(1):175–193.

Patty, John W. 2008. "Equilibrium Party Government." *American Journal of Political Science* 52(3):636–655.

Patty, John W., Constanze F. Schibber, Elizabeth Maggie Penn and Brian F. Crisp. 2019. "Valence, Elections, and Legislative Institutions." *American Journal of Political Science* 63(3):563–576.

Patty, John W. and Elizabeth Maggie Penn. 2019. "Are Moderates Better Representatives than Extremists? A Theory of Indirect Representation." *American Political Science Review* 113(3):743–761.

Pearson, Kathryn. 2015. *Party Discipline in the U.S. House of Representatives*. University of Michigan Press.

Rohde, David W. 1991. *Parties and leaders in the postreform house / David W. Rohde*. University of Chicago Press Chicago.

Skocpol, T. and V. Williamson. 2012. *The Tea Party and the Remaking of Republican Conservatism*. Oxford University Press.

Snyder, James M. and Michael M. Ting. 2002. "An Informational Rationale for Political Parties." *American Journal of Political Science* 46(1):90–110.

Snyder, James M. and Michael M. Ting. 2003. "Roll Calls, Party Labels, and Elections." *Political Analysis* 11(4):419–444.

Thomsen, Danielle M. 2017. "Joining Patterns Across Party Factions in the US Congress." *The Forum* 15(4):741.

A Theory of Intra-Party Factions and Electoral Accountability: Supplementary Materials

Contents

A Extensions

A.1 Primary Elections

In the baseline model, the voter only participates in a general election between the incumbent and a random challenger from the opposite party. In this extension, consider a primary stage (before the general election) in which the voter also decides between the incumbent and a random challenger from the same party.

Lemma 3: For districts $y < 0$, the primary voter's reelection decision can be described as follows:

- If $\mu(z|f, v) \leq -\frac{1}{2}$, then the voter reelects if and only if

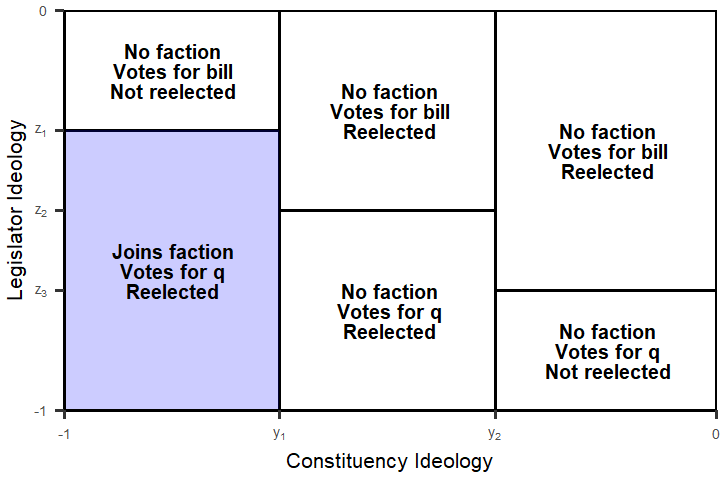
$$y \leq \frac{2\mu(z|f, v) + 2}{3}$$

- If $\mu(z|f, v) > -\frac{1}{2}$, then the voter reelects if and only if

$$y \geq \frac{2\mu(z|f, v) + 1}{3}$$

Incumbents who signal in a more "extreme" direction get reelected in the primary, if they represent districts that are sufficiently extreme. This result, captured in the first part of Lemma 3, is similar to the result in Lemma 1 in the sense that extreme incumbents in moderate districts get kicked out of office. The difference is that unlike in the general election, where all incumbents can expect to get reelected in sufficiently extreme constituencies, incumbents who signal as more moderate in these districts will face a tougher time in their primary. Thus, when the incumbent is perceived as less extreme in expectation, the primary voter has to be sufficiently moderate, as the condition in the second part of Lemma 3 indicates, in order to favor reelection. A fitting example here is the number of Republicans who

Figure A.4: Extreme Party Factions with Primaries ($q = -1$)



were primaried out of their districts during the Obama Administration because they were perceived as moderate "RINOs," or Republicans In Name Only.[18] These Republicans were ultimately swapped out for more conservative replacements.

Proposition 4: When $q < b$, a larger interval of legislators join a faction and vote in favor of the status quo. When $b < q$, the same interval of legislators do so.

The fundamental change this makes to the equilibria in Propositions 1 and 2 is that in very liberal districts, since those who are perceived as moderate are thrown out of office, a larger interval of incumbents are incentivized to send the more "extreme" signal. In the case of a conservative shift in the status quo, this means that a larger continuum of incumbents choose to join a faction and vote against the status quo in liberal districts (and those that do not lose their primary). This is described in Proposition 5 and shown visually in Figure A.4. In the case of a liberal shift in the status quo, we attain the same result, substantively speaking, as before, in the sense that the same-sized interval of legislators join a faction, though in a potentially larger interval of districts.

[18]The Hill, "Republicans on notice: Get ready for 2012 primary challenges," November 17, 2010

A.2 Insincere Votes and Non-Connected Coalitions

An increasingly common phenomenon in the U.S. Congress is the presence of non-connected voting coalitions where legislators at the ideological extremes vote against the majority party and legislators in the middle are in favor. The model can help explain this phenomenon. For the purpose of this extension, I retain the variant explored in the previous section that includes primary elections in addition to a general election.

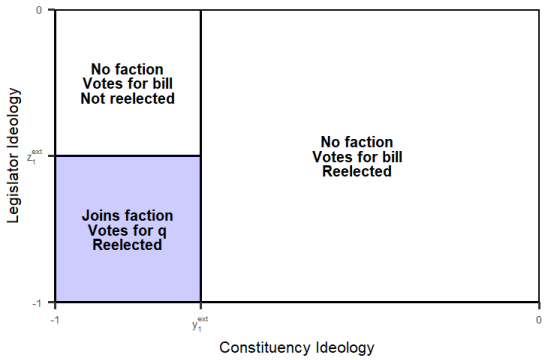
Proposition 5: If $q \geq b$, $\frac{b+q}{2} \geq 0$, $x_f \leq b - q - \frac{1}{2}$, $(q-b)^2 + (x_f)^2 + 2(q-b)b \geq w \geq (1+x_f)^2 + 2(q-b)(1+b) + (q-b)^2$, incumbents $[-1, z_1^{ext}]$ in districts $[-1, y_1^{ext}]$ vote for the status quo, and all other incumbents in the majority party vote for the bill.

This equilibrium is described in Proposition 6 and captured in Figure A.5. In the majority party, shown in panel (a), almost all legislators vote for the agenda because they are acting sincerely and voting with the party is on the equilibrium path (and leads to winning reelection). The exceptions are progressive incumbents in ideologically liberal constituencies, who defect from the agenda. The equilibrium behavior of these progressive Democrats is justified because reelection benefits are sufficiently high that it overwhelms the policy loss they incur by voting for the status quo over the party agenda (despite the party agenda being closer to their ideal points). In this perverse case, voting sincerely would lead to losing their primary election; thus, incumbents take advantage of the opportunity to signal their preferences to their constituencies and vote against their party.

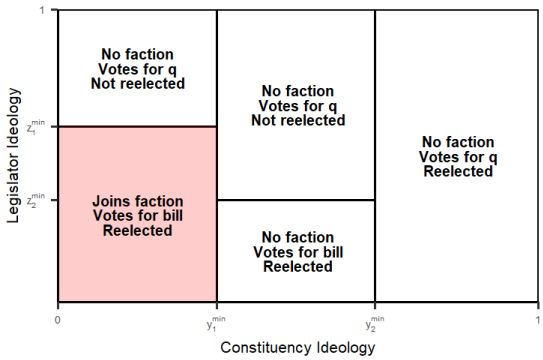
In the minority party, moderate Republicans join a faction to vote with the majority party. In this case, the equilibrium logic is the same as in Proposition 2, except the bill and status quo are flipped (since members of the minority party incur the cost of party discipline c when voting for the bill). Joining a faction in this case mitigates the cost of party discipline and garners reelection for incumbents. Those who are sufficiently conservative and prefer to vote for the status quo lose reelection.

Figure A.5: Non-Connected Voting Coalitions

(a) Majority Party ($q \geq b$)



(b) Minority Party ($q \geq b$)



One example of this equilibrium is the vote to pass President Biden's proposed infrastructure bill. On Friday, November 5, 2021, the House passed the bill on a 228-206 vote, with six Democrats voting against the bill (and 13 Republicans voting in support). The six Democrats included progressive lawmakers, like Rep. Alexandria Ocasio-Cortez (D-NY) and Rep. Ilhan Omar (D-MN), who voted "no" on the legislation because they wanted the larger Build Back Better plan to be paired with the infrastructure bill. These Democrats are apt examples of the equilibrium in panel (a) of Figure A.5. In panel (b), many of the Republicans who voted with the majority party were moderates who represented electorally competitive or moderate constituencies.

A.3 Party Welfare with Pivotal Thresholds

In the body of the paper, I detailed party welfare results using the first welfare function W_{P1}. In this section, I show the results using the second welfare function, W_{P2}. This second set of results covers similar intuitions as before. As noted above, the second welfare function assumes that parties care specifically about the threshold cases, in which a bill garners at least 50 percent of legislators' votes, and at least 50 percent of the majority party's incumbents win reelection.

To solve this, we imagine a discrete version of the model, in which there are m incumbents

in the minority party and M incumbents in the majority party, with $m < M$ and $M + m$ odd. One can define the total number of "pivotal" legislators as K. I assume one can arrange the incumbents by district as follows:

$$y_1 \leq y_2 \leq ... \leq y_M \leq 0 \leq y_{M+1} \leq ... \leq y_{M+m}$$

In other words, minority party (Republican) incumbents represent conservative districts with ideal points $y > 0$, whereas majority party (Democratic) incumbents represent liberal districts with ideal points $y < 0$.

A.3.1 Extreme Factions

First consider the extremist factions equilibrium. Note that in this equilibrium, all minority party members vote for the majority party's agenda. Suppose that extremist factions are more restrictive in their recruitment. In order for there to be a change in party welfare, we must have $T \geq \frac{M+m+1}{2}$ incumbents in the interval of districts $[-1, y_1^{ext}]$. In this case, the probability that the outcome of the roll-call vote changes between the factions equilibrium and the NF equilibrium is

$$\sum_{L=\frac{M+m+1}{2}}^{T} \sum_{K=L+1-\frac{M+m+1}{2}}^{L} \frac{T!}{(L-K)!K!(T-L)!} p^K q^{(L-K)} (1-p-q)^{(T-L)}$$

where $p = z_1^{NF,ext} - z_1^{ext}$, $q = z_1^{ext} + 1$ and K denotes the number of legislators needed to switch the outcome of the vote, for a given L, which denotes the size of the winning coalition in the factions equilibrium. Note that if the extremist faction is more restrictive in their recruitment, there cannot be any change in majority status after the election.

Now consider the case where the extremist factions are less restrictive in their recruitment. In order for there to be a change in party welfare, we need to have $T > \frac{M+m+1}{2}$ in the interval of districts $[-1, y_1^{ext}]$. In this case, the probability that the outcome of the roll-call

vote changes between the factions equilibrium and the NF equilibrium is

$$\sum_{L=\frac{M+m+1}{2}}^{T} \sum_{K=L+1-\frac{M+m+1}{2}}^{L} \frac{T!}{(L-K)!K!(T-L)!} p^K q^{(L-K)} (1-p-q)^{(T-L)}$$

where $p = z_1^{ext} - z_1^{NF,ext}$, $q = z_1^{NF,ext} + 1$, K denotes the number of legislators needed to switch the outcome of the vote, and L denotes the number of legislators in the winning coalition.

In order for there to be a change in majority status, we need to have $S < \frac{M+m+1}{2}$ in districts $[-1, y_{ext}^{NF}]$ and $T > \frac{M+m+1}{2} - S$ incumbents in the interval of districts $[y_{ext}^{NF}, y_2^{ext}]$. The probability that majority status changes is then

$$\sum_{L=\frac{M+m+1}{2}-S}^{T} \sum_{K=L+1-(\frac{M+m+1}{2}-S)}^{L} r^K s^{L-K} (1-r-s)^{T-L}$$

where $r = z_1^{ext} - z_1^{NF,ext}$, $s = z_1^{ext} + 1$. Thus, for $T \leq \frac{M+m+1}{2}$ in the interval of districts $[-1, y_1^{ext}]$, the factions equilibrium has a weakly positive effect on party welfare. Otherwise, the factions equilibrium has a negative effect on party welfare.

A.3.2 Moderate Factions

Now consider the moderate factions equilibrium, where $q > b$. Suppose that $S < \frac{M+m+1}{2}$ incumbents represent districts in the interval $[-1, y_1^{mod}]$. When moderate factions are more ideologically exclusive, the probability that the party loses majority status (when $T > M - S$ incumbents represent districts $[y_2^{mod}, 0]$) is

$$\sum_{L=M-S}^{T} \sum_{K=L+1-(M-S)}^{L} r^K s^{L-K} (1-r-s)^{T-L}$$

where $r = z_3^{mod} - z_3^{NF,mod}$ and $s = z_3^{NF,mod} + 1$.

The probability that factions affect when the party's agenda passes (when $T > M - S$)

is

$$\sum_{L=M-S}^{T} \sum_{K=L+1-(M-S)}^{L} r^K s^{L-K}(1-r-s)^{T-L}$$

As such, there is a net positive welfare gain for parties when $\alpha \geq 1$, and no change in welfare for $T \leq M - S$ in electorally competitive districts.

A.4 Endogenous Cost of Party Discipline

In this section, we endogenize the cost of party discipline c in the context of the two equilibria featured above. In this extension, I assume that the party maximizes one of the following two utility functions:

$$U_{P1} = \int \int (\mathbf{1}(v = b) + \alpha mr) dy dz$$

$$U_{P2} = Pr(\text{Bill Passes}) + \alpha Pr(\text{Majority of Seats Held by Party})$$

Note that this mimics the party's welfare function from earlier. As such, parties seek to encourage incumbents to vote in favor of their agenda, and they also want to reelect as many of their members as possible.

One should make a few additional notes here. First, I assume that parties do not observe the ideal points of the legislators, but they do observe the district y and party affiliation of the incumbent. In particular, I assume that $z|y \sim U[0, 1]$ for Republican incumbents and $z|y \sim U[-1, 0]$ for Democratic incumbents. Second, I relax the assumption that incumbents who lose reelection do not face any cost of party discipline. Finally, while majority party defections implicitly carry the same cost as minority party defections in the model, I assume in this extension that leaders can only control the cost of defections within their own party;

in other words, there are separate costs c_1 and c_0 for the majority and minority parties, respectively.

In the context of the extremist factions equilibrium, the optimal thing for the party to do is to increase the cost c_1 of party discipline to the point where all legislators, outside of the faction, vote for the party's agenda. In the context of the equilibrium, all of the incumbents in electorally competitive districts are motivated to vote in favor of the bill if $c_1 = b^2 - q^2 + 2(b-q) - w$. In addition, some incumbents will vote for the status quo in the intermediately moderate districts $[y_1^{ext}, y_2^{ext}]$ if $c_1 < 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)})$. Thus, the optimal cost is the maximum of these two quantities.

If the cost of party discipline is sufficiently high, there is a pooling equilibrium in moderate districts, where all legislators get reelected. As a result, not only would there be a greater proportion of legislators voting in favor of the party's agenda, but no legislators would lose reelection. In other words, both components of the party's utility function move in a positive direction with this strategy. Note that because this strategy yields an outcome where all incumbents (outside the faction) vote for the bill and get reelected, this optimal strategy holds for both utility functions described at the beginning of this section.

Proposition 6: For both utility functions, in the extremist factions equilibrium, $c_1^* = max\{b^2 - q^2 + 2(b-q) - w, 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)}\}$. In the moderate factions equilibrium, for function U_{P1}:

- If $\alpha < 1$, then $c_1^* = w$.
- If $\alpha \geq 1$, then $c_1^* = max\{b^2 - q^2, 0\}$.

For function U_{P2} (where M represents the number of majority party incumbents and m represents the number of minority party incumbents):

- If there exist $S < \frac{M+m+1}{2}$ incumbents in districts $[-1, y_1^{mod}]$, $T > M - S$ incumbents in the remaining districts, and $\alpha < 1$, then $c_1^* = w$. If $\alpha \geq 1$, then $c_1^* = max\{b^2 - q^2, 0\}$.

- If there exist $S > \frac{M+m+1}{2}$ incumbents in districts $[-1, y_1^{mod}]$, then there is no change in party welfare when c_1 changes.

In the moderate factions equilibrium, there is a straightforward tradeoff when the party increases c_1. When the cost of party discipline gets larger, more incumbents vote for the party's bill, but the same incumbents lose reelection because they are sending the extreme signal in equilibrium. For w sufficiently small, this only affects the most moderate districts, where faction membership is part of a necessary condition to win reelection. For w large enough, this also affects a more extreme set of districts as well. Since this region gets larger, at the margin, when c_1 increases, the first derivative is positive when $\alpha < 1$ and negative when $\alpha \geq 1$.

Note that these results hold for the second utility function as well, but only in the special case where there are sufficiently few incumbents in ideologically extreme districts (where the behavior is the same between the NF and factions equilibrium). The pivotal cases in the moderate factions equilibrium are those in which the legislators who take the party past the threshold, either in terms of capturing a majority of seats in the chamber or securing a majority of votes, are located in the gap between the factions and NF equilibria in electorally competitive districts.

A.5 Endogenous Faction Platform

In this section, we relax the assumption that the faction platform is exogenous. In particular, we consider an extension where the platform is constructed as the average of the ideal points of the faction members. One can note here that the fact that legislators' ideal points are distributed according to a continuous distribution makes it easier to derive the platforms. This is because an individual legislator's decision to join a faction does not, in and of itself, affect the faction platform, since each legislator ideal point has mass zero in the distribution

of legislator ideology. In this section, I derive the equilibrium platforms for Propositions 1 and 2.

Proposition 7: The equilibrium extremist factions platform is $x_f^{ext} = -1 - 2(b-q) + \sqrt{2(b-q)(2(b-q)+1+\frac{b+q}{2})}$. The equilibrium moderate factions platform is $x_f^{mod} = 2(q-b) - \sqrt{4(q-b)^2 - q^2 + b^2 + w}$.

For the extremist factions platform, as the status quo q moves closer to the bill b, there are two counteracting effects. First, the faction platform becomes more conservative, since more legislators sincerely prefer the status quo over the bill. However, the distance between b and q is smaller, which means that the legislators do not place as much weight on policy. Note that this faction platform satisfies the equilibrium conditions outlined in the body of the paper for Proposition 1.

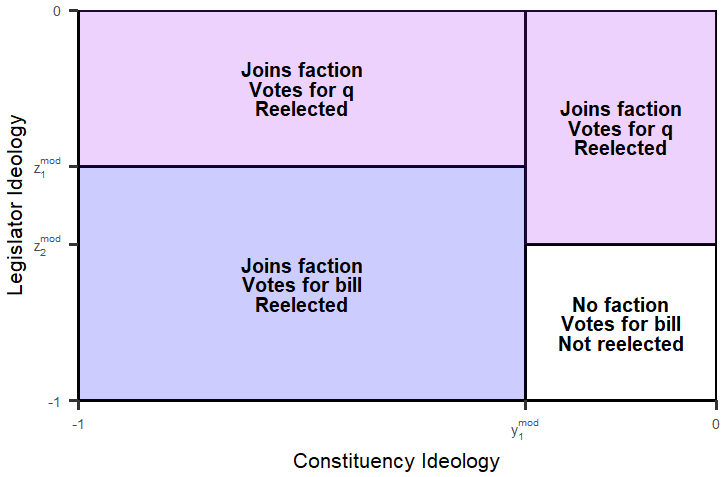
Whereas the extremist factions platform is solely a function of the bill b and the status quo q, the moderate faction platform is also a function of reelection benefits w. As reelection benefits increase, the faction platform becomes more liberal. When it comes to b and q, the same tradeoff holds for the moderate factions case as for the extremist factions case.

A.6 Multiple Factions

In this section, I relax the assumption in the model that there is only one faction in each party. Here, I explore equilibrium results with two factions in the majority party, for the case of $b < q$. The first faction's platform is denoted by x_{1f} and the second faction's platform is denoted by x_{2f}, where $x_{1f} < x_{2f}$. The two factions are shown visually in Figure A.6, with the blue faction representing faction 1 and the purple faction representing faction 2.

The first equilibrium suggests that all legislators in ideologically extreme districts join a faction, but some vote for the status quo and others vote for the party's agenda. In this

Figure A.6: Two Factions ($b < q$)



case, legislators join the first faction if and only if

$$-(z-b)^2-(z-x_{1f})^2 \geq -(z-q)^2-(z-x_{2f})^2 \Rightarrow$$

$$\Rightarrow z \leq z_{2f} = \frac{b^2-q^2+x_{1f}^2-x_{2f}^2}{2(b-q+x_{1f}-x_{2f})}$$

This equilibrium is justified if no legislators have an incentive to not join a faction and vote the same way. This requires $\sqrt{w} \geq max\{1+x_{1f}, z_{2f}-x_{1f}, x_{2f}-z_{2f}, -x_{2f}\}$. We also need legislators not to deviate in the roll-call voting stage and not to prefer for the opposite policy in the first stage. This is satisfied given the condition on w. The second equilibrium takes place in electorally competitive districts and is identical to the one illustrated in Proposition 2.

To consider an equilibrium with three factions, it is easy to see that one can assume a third faction platform x_{3f} where the faction equilibrium in the electorally competitive districts holds. In other words, one can imagine a scenario in which there are two factions in ideologically extreme districts and a third separate faction (which in Figure A.6 is also

highlighted in purple) in ideologically moderate districts. An example of this three-faction case might be the Congressional Progressive Caucus, the New Democrat Coalition, and the Blue Dog Democrats. The Congressional Progressive Caucus mostly comprises liberal incumbents in ideologically extreme districts. The New Democrat Coalition and the Blue Dog Coalition both comprise moderate Democrats (relative to the rest of the party), but the New Democrat Coalition contains a broader cross-section of Democrats than the Blue Dogs, who are mainly concentrated in conservative/electorally competitive districts.

A.7 Voter Welfare

In this section, we turn away from party welfare to consider the voter's welfare function:

$$W_{V2} = -E_z[(y-z)^2] - E_z[(y-v(z))^2]$$

Note that the first term mimics the voter's utility function from the main text. Accordingly, faction equilibria may be welfare-improving by more carefully delineating the ideological type of the incumbent to the voters, thereby allowing voters to weed out more of the dissonant types in equilibrium. The new second term represents the congruence between the incumbents' votes and their constituency's preferences. As such, factions may also be optimal for voters if they encourage incumbents to vote for the district's preferred policies. Note that these terms represent "selection" and "sanctioning" effects, respectively. In the next section, I extend this further such that voters derive greater welfare not just from their own legislators' votes but others' votes too (e.g., $W_{V3} = -E_z[(y-z)^2] - E_{\tilde{y}}E_z(y - v_{\tilde{y},z}(z))^2$).

First consider the extreme factions equilibrium in Proposition 1. As Proposition 8 states, if the midpoint between the bill and the status quo is sufficiently liberal (i.e., $\frac{b+q}{2} \in [-1, y_1^{ext}]$), then there are heterogeneous effects across districts. Districts $y \in [\frac{b+q}{2}, y_1^{ext}]$ accrue greater voter welfare from the factions equilibrium because fewer legislators vote for the status quo given the high costs from joining a faction and avoiding the cost of party discipline. On the

flip side, districts $y \in [-1, \frac{b+q}{2}]$ prefer the NF equilibrium. It is straightforward to see that if $\frac{b+q}{2} \geq y_1^{ext}$, then none of the districts $y \in [-1, y_1^{ext}]$ prefer the factions equilibrium.

Proposition 8 (Voter Welfare):

- If $z_1^{ext} \leq z_1^{NF,ext}$ (or equivalently, $x_f \leq \hat{x}_f$) and $\frac{b+q}{2} \in [-1, y_1^{ext}]$, the extreme factions equilibrium accrues greater voter welfare than the NF equilibrium in districts $y \in [\frac{b+q}{2}, y_1^{ext}]$ and accrues less welfare in districts $y \in [-1, \frac{b+q}{2}]$. If $\frac{b+q}{2} \geq y_1^{ext}$, then no district prefers the factions equilibrium.

- If $z_3^{mod} \geq z_2^{NF,mod}$ (or equivalently, $x_f \geq \tilde{x}_f$), $\frac{b+q}{2} \leq 0$, and $q - b \geq \frac{1}{2}$, the moderate factions equilibrium accrues greater voter welfare than the NF equilibrium in districts $y \in [y_2^{mod}, \hat{y}]$ and accrues less welfare in districts $y \in [\hat{y}, 0]$. If $\frac{b+q}{2} \geq 0$ or $q - b < \frac{1}{2}$, then all districts $[y_2^{mod}, 0]$ prefer the factions equilibrium.

Now consider the moderate factions equilibrium, where the midpoint $\frac{b+q}{2} \geq 0$. If faction recruitment is sufficiently restrictive, then this has a positive effect on voter welfare through both the selection and sanctioning mechanisms. First, if fewer legislators join a faction, then faction membership becomes a more precise signal of ideology, and voters (in electorally competitive districts) are better equipped to throw out dissonant types. This leads to more congruent types remaining in office after the election. Second, fewer incumbents are voting for the status quo in equilibrium; this derives policy utility for the districts these incumbents represent. If the midpoint is negative (i.e., $\frac{b+q}{2} \leq 0$), then this analysis becomes more complicated, as the policy loss from fewer representatives voting for the status quo may outweigh the representational gain from removing dissonant types from office. As such, if the distance between the bill and status quo is sufficiently large, only districts sufficiently liberal (or close to the positioning of the bill), or $y \leq \hat{y}$, receive greater welfare from the factions equilibrium.

A.8 Sanctioning Effect (Alternative Specification)

In this section, I extend the welfare analysis in the paper to consider a case in which voters derive welfare not just from their own representative's roll-call votes but others' votes too. I consider the following function:

$$W = -E_z[(y-z)^2] - E_{\tilde{y}}E_z(y - v_{\tilde{y},z}(z))^2$$

First consider the extreme factions equilibrium. If factions are sufficiently restrictive in their recruitment ($z_1^{ext} \leq z_1^{NF,ext}$ or equivalently, $x_f \leq \hat{x}_f$), then districts $y \in [-1, \frac{b+q}{2}]$ accrue less welfare (which is the same as in Proposition 8), but districts $y \in [\frac{b+q}{2}, 1]$ accrue greater welfare. This is purely driven by the sanctioning effect (and the location of the midpoint between the bill and the status quo).

Proposition 9 (Sanctioning Effect—Alternative Specification):

- If $z_1^{ext} \leq z_1^{NF,ext}$ (or equivalently, $x_f \leq \hat{x}_f$), the extreme factions equilibrium accrues greater voter welfare than the NF equilibrium in districts $y \in [\frac{b+q}{2}, 1]$ and accrues less welfare in districts $y \in [-1, \frac{b+q}{2}]$.
- If $z_3^{mod} \geq z_2^{NF,mod}$ (or equivalently, $x_f \geq \tilde{x}_f$) and $\frac{b+q}{2} \leq 0$, the moderate factions equilibrium accrues greater voter welfare than the NF equilibrium in districts $y \in [-1, \bar{y}]$ and accrues less welfare in districts $y \in [\bar{y}, 1]$, where $\bar{y} \geq \frac{b+q}{2}$. If $\frac{b+q}{2} \geq 0$, then all districts $[-1, \frac{b+q}{2}]$ prefer the factions equilibrium.

For the moderate factions equilibrium, $z_3^{mod} \geq z_2^{NF,mod}$ (or equivalently, $x_f \geq \tilde{x}_f$), in the easy case where $\frac{b+q}{2} \geq 0$, ideologically exclusive factions are optimal for voters to the left of the midpoint, and not to the right. If the midpoint is negative, then like in Proposition 8, the analysis is more complicated. Some districts to the right of the midpoints accrue less

welfare because more legislators are voting in favor of the bill, but some districts benefit from the selection effect.

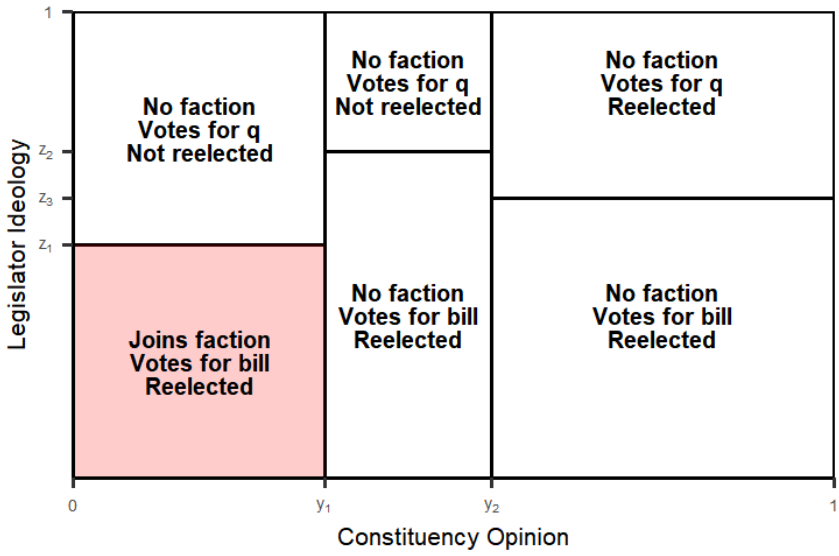
B Minority Party Factions

The agenda put forward by majority party leaders also affects the decisions of minority party members to join factions as well. In particular, the result in Proposition 10 suggests that when there is an ideological split within the minority party, there exists a faction that comprises moderate members who coalesce with the majority party to vote in favor of the proposed agenda. One should note that in the set-up of this model, minority and majority party members are assumed not to have overlapping ideal points (i.e., the most conservative Democrat is to the left of the most liberal Republican). As a result, a coalition of minority and majority party members does not exist because some minority party members share the same ideal points as some in the majority, but rather they have the same preference ranking over the bill and status quo or members of the minority may have an incentive to appear moderate to their constituencies by voting with the majority.

Proposition 10: If $q \geq b$, members $z \in [0, z_1^{min}]$ in districts $y \in [0, y_1^{min}]$ join a faction and vote for the majority party's bill, and members $z \in (z_1^{min}, 1]$ vote for the status quo.

When the bill proposed by the majority party is more liberal than the status quo, in the most moderate districts ($y \in [0, y_1^{min}]$), liberal incumbents will join a faction and vote for the party's bill, and incumbents that are sufficiently conservative will not join a faction and vote for the status quo. This is shown visually in Figure B.7. One example that fits this setting (if one flips the axes again to look at the Republican side) is during the Bush Administration, when the House was controlled by Republicans. The Blue Dog Democrats voted with Republicans on several initiatives, notably the Bush tax cuts, various national

Figure B.7: Minority Party Factions ($b = 0$, $q = 1$)



security measures like the National Commissions Act, and the Partial-Birth Abortion Act. On each of these votes, moderate Democrats representing more conservative districts sided with the majority party.

C Additional Analysis on Roll-Call Votes

In the case studies that I have drawn up, I focused on two key roll-call votes. To corroborate the analysis I provided and to link these votes to the equilibria described in the paper, I show breakdowns of the roll-call votes by legislator ideology, constituency ideology, and faction membership.

On the DHS funding measure in the first case study, one can look at Table C.1, which breaks down the vote on the DHS funding bill by membership in the House Freedom Caucus (as measured by Clarke 2020), constituency ideology (as measured by Tausanovitch and Warshaw 2013), and legislator ideology (as measured by the first-dimension DW-NOMINATE score). The first two rows show that members of the House Freedom Caucus were much more likely to vote against the DHS funding measure than other Republicans (in fact, none

of them voted in favor). Moreover, the third and fourth row show that incumbents who represented very conservative districts were also more likely to vote against the bill, and the same is true for very conservative incumbents (as seen in the fifth and sixth row). Finally, virtually all conservative legislators (about 98.3 percent) who represented conservative districts voted against the bill.

Table C.1: DHS Vote by HFC Membership, Constituency Ideology, and Legislator Ideology

Group	Proportion of Ayes	Proportion of Nays
HFC	0	1
Non-HFC	0.368	0.632
Cons. Districts (>75th percentile)	0.088	0.912
Non-cons. Districts (<=75th percentile)	0.378	0.622
Cons. Legislators (>75th percentile)	0.017	0.983
Non-cons. Legislators (<=75th percentile)	0.407	0.593
Cons. Legislators in Cons. Districts	0	1
~ (Cons. Legislators in Cons. Districts)	0.338	0.662
All Members of Congress	0.31	0.69

Similarly, on the Waxman-Markey vote, which is broken down in Table C.2, members of the Blue Dog Democrats were much more likely to vote against the legislation than non-members (54 percent versus 8.3 percent). Those in the most conservative districts were also more likely to vote against the bill, and the same is true for the most conservative legislators in the Democratic Party. Finally, this proportion is even larger (about 58.1 percent) when one considers conservative Democratic legislators who represent conservative districts.

D Relaxation of Assumptions

In the paper, I imposed the following condition: $(2+b+q)|b-q| \geq w \geq c \geq max\{b^2-q^2, 0\}$. In this section, I discuss how relaxing parts of this condition affects the equilibria in Propositions 1 and 2.

In the context of Proposition 1, if w is allowed to be sufficiently large, then all legislators

Table C.2: Waxman-Markey Vote by Blue Dog Membership, Constituency Ideology, and Legislator Ideology

Group	Proportion of Ayes	Proportion of Nays
Blue Dog	0.46	0.54
Non-BD	0.917	0.083
Cons. Districts (>75th percentile)	0.492	0.508
Non-cons. Districts	0.942	0.058
Cons. Legislators (>75th percentile)	0.531	0.469
Non-cons. Legislators	0.927	0.073
Cons. Legislators in Cons. Districts	0.419	0.581
~ (Cons. Legislators in Cons. Districts)	0.91	0.09
All Members of Congress	0.827	0.173

in districts $[y_2^{ext}, 0]$ vote in favor of the bill and win reelection. Similarly if c is sufficiently large, then all legislators in districts $[y_1^{ext}, 0]$ vote in favor of the bill and win reelection. Note that in this scenario, the factions equilibria do not have any effect on party welfare by helping to reelect incumbents (though they do continue to have effects by encouraging more defections). Relaxing the lower bound on c does not change these results.

In the context of Proposition 2, the upper bound on w ensures that at least some incumbents lose reelection on the equilibrium path in districts $[y_1^{mod}, 0]$. If one relaxes the lower bound on c, then at least some legislators in districts $[-1, y_1^{mod}]$ vote for the status quo.

E Proofs

Proof of Lemma 1: The first part of Lemma 1 is proved in the body of the paper. The second part can be shown by solving the following inequality:

$$-(y - \frac{1}{2})^2 - \frac{1}{12} \leq -(y - \mu(z|f,v))^2 - \sigma^2(z|f,v) \Rightarrow$$

$$\Rightarrow y \leq \left(\frac{1 - 3(\mu(z|f,v))^2 - 3\sigma^2(z|f,v)}{3(1 - 2\mu(z|f,v))} \right)$$

Note that it is possible to have signals in which $\sigma^2(z|f,v) > \frac{1}{12}$. One hypothetical example is a case in which incumbents $z \in [-1, -\frac{4}{5}]$ and $z \in [-\frac{1}{5}, 0]$ join a faction and all other incumbents abstain. Here, the posterior variance associated with faction members is equal to $\frac{49}{100} > \frac{1}{12}$.

For the case in which $\sigma^2(z|f,v) > \frac{1}{12}$, the same inequality above holds where $\mu(z|f,v) \leq -\frac{1}{2}$. If the variance is larger and the mean is closer to zero (than -1), then there are some cases in which the reduction in bias always outweighs the increase in the variance, particularly when $\sigma^2(z|f,v) \leq \frac{1}{3} - (\mu(z|f,v))^2$. Otherwise, one can again use the inequality above to identify the districts that would prefer to reelect the incumbent.

Proof of Lemma 2: The terms invoked in Lemma 2 are defined as follows: $y_{mod}^{NF} = \frac{1}{3} z_2^{NF,mod} \frac{z_2^{NF,mod}-1}{z_2^{NF,mod}-2}$, $y_{ext}^{NF} = \frac{1}{3} z_1^{NF,ext} \frac{z_2^{NF,ext}-1}{z_2^{NF,ext}-2}$, $z_2^{NF,mod} = \frac{b+q}{2} - \frac{w}{2(q-b)}$, $z_2^{NF,ext} = \frac{b+q}{2} - \frac{w}{2(b-q)}$, $z_1^{NF,ext} = \frac{b+q}{2} + \frac{c}{2(b-q)}$. The proofs for these equilibria are analogous to those described in the proofs for Propositions 1 and 2.

Proof of Lemma 3: The first part of Lemma 3 can be shown from the inequality

$$-(y + \frac{1}{2})^2 - \frac{1}{12} \leq -(y - \mu(z|f,v))^2 - \frac{(2(\mu(z|f,v)+1))^2}{12} \Rightarrow$$

$$\Rightarrow y \leq \frac{2\mu(z|f,v) + 2}{3}$$

The RHS is always positive, since $\mu(z|f,v) \geq -1$.

The second part of Lemma 3 can be shown from the inequality

$$-(y + \frac{1}{2})^2 - \frac{1}{12} \leq -(y - \mu(z|f,v))^2 - \frac{4\mu(z|f,v)^2}{12} \Rightarrow$$

$$\Rightarrow y \geq \frac{2\mu(z|f,v) + 1}{3}$$

Proof of Proposition 1: The full equilibrium is described below:

1. Incumbents $-1 \leq z \leq z_1^{ext}$ in districts $[-1, y_1^{ext}]$ join a faction and vote in favor of the status quo, and incumbents $z > z_1^{ext}$ do not.
2. If $c \leq \hat{c} = 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)})$:
   - Incumbents $z \leq z_2^{ext}$ in districts $[y_1^{ext}, y_2^{ext})$ vote in favor of the status quo, and incumbents $z > z_2^{ext}$ vote in favor of the party's bill.

- Incumbents $z \leq z_3^{ext}$ vote in favor of the status quo and do not get reelected. Incumbents $z > z_3^{ext}$ vote in favor of the party's bill and do get reelected.

3. If $c > \hat{c} = 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)})$:

   - Incumbents $z \leq z_3^{ext}$ in districts $y \in [y_1^{ext}, 0]$ vote in favor of the status quo and do not get reelected. Incumbents $z > z_3^{ext}$ vote in favor of the party's bill and do get reelected.

First, we ascribe values to the thresholds in this equilibrium (which are also noted in Proposition 1). We have $z_1^{ext} = x_f + q - b + \sqrt{2(b-q)(b-x_f)}$, $z_2^{ext} = \frac{b+q}{2} - \frac{c}{2(b-q)}$, $z_3^{ext} = \frac{b+q}{2} - \frac{w}{2(b-q)}$, $y_1^{ext} = \frac{1}{3} z_1^{ext} (\frac{1-z_1^{ext}}{2-z_1^{ext}})$, and $y_2^{ext} = \frac{1}{3} z_2^{ext} (\frac{1-z_2^{ext}}{2-z_2^{ext}})$. The upper threshold for x_f is $\underline{x}_f^1 = -1 + \sqrt{(1+b)^2 + (1+q)^2}$ and if $b+q \geq 0$, it is equal to $\underline{x}_f^1 = \sqrt{b^2 - q^2}$. The lower bound on w is $\tilde{w} = -2(\sqrt{2(b-q)(b-x_f)} + x_f - \frac{3b-q}{2})(b-q)$.

The voters' posterior mean in $y \in [-1, y_1^{ext}]$ is:

$$\mu(z) = \begin{cases} \frac{1}{2}(-1 + z_1^{ext}) & f = 1 \text{ and } v = q \\ \frac{1}{2}(z_1^{ext}) & f = 0 \text{ and } v = b \\ \frac{1}{2} & f = 1 \text{ and } v = b \text{ or } f = 0 \text{ and } v = q \end{cases}$$

The voters' posterior variance is:

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(-1 - z_1^{ext})^2 & f = 1 \text{ and } v = q \\ \frac{(z_1^{ext})^2}{12} & f = 0 \text{ and } v = b \\ \frac{1}{12} & f = 1 \text{ and } v = b \text{ or } f = 0 \text{ and } v = q \end{cases}$$

Those who join a faction prefer to do so when

$$U(z, f = 1, v = q) = -(z - x_f)^2 - (z - q)^2 \geq -(z - b)^2 = U(z, f = 0, v = b)$$

which is equivalent to $z \in [x_f + q - b - \sqrt{2(b-q)(b-x_f)}, x_f + q - b + \sqrt{2(b-q)(b-x_f)}]$.

One can note that the lower bound $x_f + q - b - \sqrt{2(b-q)(b-x_f)}$ is less than -1 when $(1+q)^2 + (1+x_f)^2 \leq (1+b)^2$, which is true under the condition on x_f. It is always less than 0, given the conditions $q < b$ and $x_f \in [-1, 0]$. Moreover, the higher bound z_1^{ext} is weakly greater than -1, whenever $x_f \leq -1 + \sqrt{(1+b)^2 + (1+q)^2}$, and it is less than 0 when $b^2 - q^2 \leq (x_f)^2$, which is true if $b + q \leq 0$ or $x_f \leq \sqrt{b^2 - q^2}$.

Those who join a faction prefer to vote for the status quo in the later stage when

$$U(z, f = 1, v = q) = -(z - q)^2 + w \geq -(z - b)^2 = U(z, f = 1, v = b)$$

It is easy to see that this condition is always satisfied since this is a less strict condition on z than the earlier one which we used to derive z_1^{ext}.

Those who do not join a faction prefer to vote for the bill when

$$U(z, f = 0, v = b) = -(z - b)^2 + w \geq -(z - q)^2 = U(z, f = 0, v = q)$$

which is equivalent to $z \geq \frac{b+q}{2} - \frac{w}{2(b-q)}$. One can show that this threshold is lower than z_1^{ext}. The condition for this is

$$\frac{b+q}{2} - \frac{w}{2(b-q)} \leq q - b + x_f + \sqrt{2(b-q)(b-x_f)}$$

which yields the condition $w \geq \tilde{w} = -2(\sqrt{2(b-q)(b-x_f)} + x_f - \frac{3b-q}{2})(b-q)$.

If $c \leq 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)})$, the voter's posterior mean in $y \in [y_1^{ext}, y_2^{ext}]$ is

$$\mu(z) = \begin{cases} \frac{1}{2}(-1 + z_2^{ext}) & f = 0 \text{ and } v = q \\ \frac{1}{2} z_2^{ext} & f = 0 \text{ and } v = b \\ \frac{1}{2} & f = 1 \end{cases}$$

The voters' posterior variance is:

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(-1 - z_2^{ext})^2 & f = 0 \text{ and } v = q \\ \frac{1}{12}(z_2^{ext})^2 & f = 0 \text{ and } v = b \\ \frac{1}{12} & f = 1 \end{cases}$$

A legislator prefers to vote in favor of the status quo when

$$U(z, f = 0, v = q) = -(z - q)^2 - c \geq -(z - b)^2 = U(z, f = 0, v = b)$$

which is true for $z \leq \frac{b+q}{2} - \frac{c}{2(b-q)}$. There is no incentive to deviate to joining a faction when

$$U(z, f = 1, v = q) = -(z-q)^2 - (z-x_f)^2 \leq -(z-q)^2 - c + w = U(z, f = 0, v = q)$$

which simplifies to $-(z-x_f)^2 \leq -c+w$, which is always true (since $w \geq c$). There is no incentive to deviate for those who are voting for the bill when

$$U(z, f = 1, v = q) = -(z-q)^2 - (z-x_f)^2 \leq -(z-b)^2 + w = U(z, f = 0, v = b)$$

which is true for incumbents above the threshold z_2^{ext} (because $c \leq w$). It is trivial that there is no deviation to joining a faction and voting for the bill.

Finally, we consider districts $y \in [y_2^{mod}, 0]$. The voter's posterior mean is

$$\mu(z) = \begin{cases} \frac{1}{2}(-1 + z_3^{mod}) & f = 0 \text{ and } v = q \\ \frac{1}{2} z_3^{mod} & f = 0 \text{ and } v = b \\ \frac{1}{2} & f = 1 \end{cases}$$

The voters' posterior variance is:

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(-1 - z_3^{mod})^2 & f = 0 \text{ and } v = q \\ \frac{1}{12}(z_3^{mod})^2 & f = 0 \text{ and } v = b \\ \frac{1}{12} & f = 1 \end{cases}$$

The legislators prefer to vote for the status quo when

$$U(z, f = 0, v = q) = -(z-q)^2 \geq -(z-b)^2 + w = U(z, f = 0, v = b)$$

which is true for $z \leq \frac{b+q}{2} - \frac{w}{2(b-q)}$. Those who vote for the bill have no such incentive to deviate when

$$U(z, f = 0, v = b) = -(z-b)^2 + w \geq -(z-q)^2 - (z-x_f)^2 = U(z, f = 1, v = q)$$

which is always true for any z that votes for the bill in equilibrium.

Next, we consider what happens when $c > 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)})$.

The voter's posterior mean is

$$\mu(z) = \begin{cases} \frac{1}{2}(-1 + z_3^{mod}) & f = 0 \text{ and } v = q \\ \frac{1}{2} z_3^{mod} & f = 0 \text{ and } v = b \\ \frac{1}{2} & f = 1 \end{cases}$$

The voters' posterior variance is:

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(-1 - z_3^{mod})^2 & f = 0 \text{ and } v = q \\ \frac{1}{12}(z_3^{mod})^2 & f = 0 \text{ and } v = b \\ \frac{1}{12} & f = 1 \end{cases}$$

The legislators prefer to vote for the status quo when

$$U(z, f = 0, v = q) = -(z - q)^2 \geq -(z - b)^2 + w = U(z, f = 0, v = b)$$

which is true for $z \leq \frac{b+q}{2} - \frac{w}{2(b-q)}$. There is no incentive to join a faction for those who are voting for the status quo because there is no cost of party discipline exerted. Those who vote for the bill have no such incentive to deviate when

$$U(z, f = 0, v = b) = -(z - b)^2 + w \geq -(z - q)^2 - (z - x_f)^2 = U(z, f = 1, v = q)$$

which is always true for any z that votes for the bill in equilibrium.

Proof of Proposition 2: The full equilibrium can be described as follows:

- In districts $y \in [y_2^{mod}, 0]$, incumbents $z \in [z_3^{mod}, 0]$ join a faction and vote for the status quo. Incumbents $z < z_3^{mod}$ vote for the party's bill and lose reelection.
- If $c < \frac{q-3b}{q-b} + 2(q-b)x_f + w - 2(q-b)\sqrt{2(q-b)(x_f - b) + w}$:
  - In districts $y \in [y_1^{mod}, y_2^{mod})$, incumbents $z \geq z_2^{mod}$ vote in favor of the status quo, whereas incumbents $z < z_2^{mod}$ vote in favor of the party's bill and lose reelection
  - In districts $y \in [-1, y_1^{mod}]$, all incumbents vote in favor of the bill and get reelected.
- If $c \geq \frac{q-3b}{q-b} + 2(q-b)x_f + w - 2(q-b)\sqrt{2(q-b)(x_f - b) + w}$:

– In districts $y \in [-1, y_2^{mod}]$, all incumbents vote in favor of the bill and get reelected.

We have $z_2^{mod} = \frac{b+q}{2} - \frac{w-c}{2(q-b)}$, $z_3^{mod} = q - b + x_f - \sqrt{2(q-b)(x_f - b) + w}$, $y_2^{mod} = \frac{1}{3} z_3^{mod} (\frac{1-z_3^{mod}}{2-z_3^{mod}})$, and $y_1^{mod} = \frac{1}{3} z_2^{mod} (\frac{1-z_2^{mod}}{2-z_2^{mod}})$. We need $x_f \geq max\{b - \frac{w}{2(q-b)}, -\sqrt{b^2 - q^2 + w}\}$.

If $c < \frac{q-3b}{q-b} + 2(q-b)x_f + w - 2(q-b)\sqrt{2(q-b)(x_f - b) + w}$, the voters' posterior mean in $y \in [y_1^{ext}, y_2^{ext}]$ is:

$$\mu(z) = \begin{cases} \frac{1}{2}(-1 + z_2^{ext}) & f = 0 \text{ and } v = b \\ \frac{z_2^{ext}}{2} & f = 0 \text{ and } v = q \\ \frac{1}{2} & f = 1 \end{cases}$$

The voters' posterior variance is:

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(-1 - z_2^{ext})^2 & f = 0 \text{ and } v = b \\ \frac{(z_2^{ext})^2}{12} & f = 0 \text{ and } v = q \\ \frac{1}{12} & f = 1 \end{cases}$$

Incumbents in these districts vote in favor of the bill when

$$U_{z,f=0,v=b} = -(z-b)^2 \geq -(z-q)^2 + w - c = U_{z,f=0,v=q}$$

which is equivalent to $z \leq \frac{b+q}{2} - \frac{w-c}{2(q-b)}$.

There is no incentive for the incumbents voting for the status quo to join a faction when

$$U_{z,f=0,v=q} = -(z-q)^2 + w - c \geq -(z-q)^2 - (z-x_f)^2 = U_{z,f=1,v=q}$$

which is always true since this simplifies to $w - c \geq -(z - x_f)^2$ (and $w \geq c$). There is no incentive to deviate for the members voting for the majority party bill because they (sincerely) prefer the bill over the status quo. There is no incentive to join a faction and vote in favor of the bill because

$$U_{z,f=0,v=q} = -(z-q)^2 + w - c \geq -(z-b)^2 - (z-x_f)^2 = U_{z,f=1,v=b}$$

which yields a threshold that is higher than z_2^{ext}. It is trivial that there is no incentive to deviate for those who vote for the bill in equilibrium.

If $c \geq \frac{q-3b}{q-b} + 2(q-b)x_f + w - 2(q-b)\sqrt{2(q-b)(x_f - b) + w}$, the equilibrium in $y \in [y_1^{ext}, y_2^{ext}]$ is the

same as in $[-1, y_1^{ext}]$. In districts $y \in [y_1^{ext}, 0]$, the voters' posterior mean is

$$\mu(z) = \begin{cases} \frac{1}{2}(-1 + z_3^{ext}) & f = 0 \text{ and } v = b \\ \frac{z_3^{ext}}{2} & f = 1 \text{ and } v = q \\ \frac{1}{2} & f = 0 \text{ and } v = q \text{ or } f = 1 \text{ and } v = b \end{cases}$$

The voters' posterior variance is:

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(-1 - z_3^{ext})^2 & f = 0 \text{ and } v = b \\ \frac{(z_3^{ext})^2}{12} & f = 1 \text{ and } v = q \\ \frac{1}{12} & f = 0 \text{ and } v = q \text{ or } f = 1 \text{ and } v = b \end{cases}$$

Incumbents in these districts vote in favor of the bill (instead of joining a faction and voting for the status quo) when

$$U_{z,f=0,v=b} = -(z-b)^2 \geq -(z-q)^2 - (z-x_f)^2 + w = U_{z,f=1,v=q}$$

which is equivalent to $z = z_3^{mod} \leq q - b + x_f - \sqrt{2(q-b)(x_f - b) + w}$. The term under the square root is positive if $x_f \geq b - \frac{w}{2(q-b)}$. z_3^{mod} is less than zero when $w \geq q^2 - b^2 + (x_f)^2$.

Incumbents who join a faction do not have an incentive to vote for the bill in a later round when

$$U_{z,f=1,v=q} = -(z-q)^2 + w \geq -(z-b)^2 = U_{z,f=0,v=b}$$

which is equivalent to $z \geq \frac{q+b}{2} - \frac{w}{2(q-b)}$. By inspection, this condition is less strict than the one that determines faction membership.

Similarly, incumbents who do not join a faction do not have an incentive to vote for the status quo when

$$U_{z,f=0,v=b} = -(z-b)^2 \geq -(z-q)^2 - w = U_{z,f=0,v=q}$$

which is true when $z \leq \frac{b+q}{2} + \frac{w}{2(q-b)}$. If $b + q \geq 0$, then this is trivially satisfied. If $b + q \leq 0$, then that means $b^2 - q^2 \geq 0$. We already assume that $w \geq max\{b^2 - q^2, 0\}$, so the condition on z translates to $z \leq 0$, which is always true for Democratic incumbents.

Proof of Proposition 3: If $z_1^{ext} > z_1^{NF,ext}$, then the extremist factions equilibrium accrues greater party welfare if and only if

$$\alpha(z_2^{NF,ext}+1)(y_2^{ext}-y_1^{NF,ext})-(z_1^{ext}+1)(y_2^{ext}+1)+(y_1^{NF,ext}+1)(z_1^{NF,ext}+1)+(y_2^{ext}-y_1^{NF,ext})(z_2^{NF,ext}+1)\geq 0 \Rightarrow$$

$$\Rightarrow (1+\alpha)(z_2^{NF,ext}+1)(y_2^{ext}-y_1^{NF,ext})\geq (z_1^{ext}+1)(y_2^{ext}+1)-(y_1^{NF,ext}+1)(z_1^{NF,ext}+1)\Rightarrow$$

$$\Rightarrow 1+\alpha \geq \frac{(z_1^{ext}+1)(y_2^{ext}+1)-(y_1^{NF,ext}+1)(z_1^{NF,ext}+1)}{(z_2^{NF,ext}+1)(y_2^{ext}-y_1^{NF,ext})}$$

For the moderate factions equilibrium, one can derive an analogous condition to the one above.

Proof of Proposition 4: The threshold for the extreme party factions in the absence of primaries is $q-b+x_f+\sqrt{2(b-q)(b-x_f)}$, and the corresponding threshold with primaries is $q-b+x_f+\sqrt{2(b-q)(b-x_f)+w}$. The threshold for moderate party factions is the same, with and without primaries.

Proof of Proposition 5: Since the equilibrium on the Republican side is analogous to the results in the Minority Party Factions section, I focus here on the proof for the Democratic faction. For districts $[-1, y_1^{ext}]$, the voters' posterior mean is

$$\mu(z)=\begin{cases}\frac{-1+z_1^{ext}}{2} & f=1 \text{ and } v=q\\ \frac{z_1^{ext}}{2} & f=0 \text{ and } v=b\\ \frac{1}{2} & \text{otherwise}\end{cases}$$

and the voters' posterior variance is

$$\sigma^2(z)=\begin{cases}\frac{1}{12}(1+z_1^{ext})^2 & f=1 \text{ and } v=q\\ \frac{(z_1^{ext})^2}{12} & f=0 \text{ and } v=b\\ \frac{1}{12} & \text{otherwise}\end{cases}$$

Incumbents join a faction when

$$-(z-x_f)^2-(z-q)^2\geq -(z-b)^2\Rightarrow$$

$$\Rightarrow z \in [x_f + q - b - \sqrt{2(b-q)(b-x_f)+w}, x_f + q - b + \sqrt{2(b-q)(b-x_f)+w}]$$

The lower threshold is less than -1 when $w \geq (1+x_f)^2 + 2(q-b)(1+b) + (q-b)^2$, and the upper threshold z_1^{ext} is less than 0 if $w \leq (q-b)^2 + (x_f)^2 + 2(q-b)b$. In order for incumbents who abstained from faction membership to not have an incentive to deviate by voting for the status quo later on, we need $\frac{b+q}{2} \geq 0$. The condition on x_f is required so that faction members are signaling as extreme in equilibrium.

Proof of Proposition 6: The cost of party discipline that causes all incumbents in electorally competitive districts to vote for the bill satisfies:

$$\frac{b+q}{2} - \frac{w + c_1^*}{2(b-q)} = -1 \Rightarrow$$

$$\Rightarrow c_1^* = b^2 - q^2 + 2(b-q) - w$$

Moreover, the cost of party discipline that causes the most incumbents in the middle group of districts $[y_1^{ext}, y_2^{ext}]$ to vote for the bill satisfies:

$$q - b + x_f + \sqrt{2(b-q)(b-x_f)} = \frac{b+q}{2} - \frac{c}{2(b-q)}$$

which is $c_1^* = 2(b-q)(\frac{3b-q}{2} - x_f - \sqrt{2(b-q)(b-x_f)})$. Since party utility is increasing in c in this case, the optimal c is equal to the maximum of these two thresholds.

In the moderate factions equilibrium, the first derivative with respect to c is equal to:

$$(1-\alpha)\frac{\partial}{\partial c}((y_2^{mod} - y_1^{mod})(z_2^{mod}+1) + (-y_2^{mod})(z_3^{mod}+1))$$

The second multiplied term is always positive, so the sign of the first derivative depends on whether $\alpha \geq 1$.

Proof of Proposition 7: To derive the equilibrium platform for the extremist factions equilibrium, one has to rearrange the following equation:

$$\frac{1}{2}(x_f^{ext} + q - b + \sqrt{2(b-q)(b-x_f^{ext})}) \Rightarrow$$

$$\Rightarrow x_f^{ext} = -1 - 2(b-q) + \sqrt{2(b-q)(2(b-q)+1+\frac{b+q}{2})}$$

For the moderate factions equilibrium, the factions platform satisfies the following equation:

$$\frac{1}{2}(q - b + x_f^{mod} - \sqrt{2(q-b)(x_f^{mod}-b)+w}) = x_f^{mod} \Rightarrow$$

$$\Rightarrow x_f^{mod} = 2(q-b) - \sqrt{4(q-b)^2 + b^2 - q^2 + w}$$

Proof of Proposition 8: The first part of Proposition 8 is straightforward to derive. The second part can be derived by solving the following inequality:

$$-(y-q)^2(-z_3^{mod}) - (y-b)^2(z_3^{mod}+1) - (-(y-q)^2(-z_2^{NF,mod}) - (y-b)^2(z_2^{NF,mod}+1)) - (y-\frac{z_3^{mod}}{2})^2 - \frac{(z_3^{mod})^2}{12} - (-(y-\frac{z_2^{mod}}{2})^2 - \frac{(z_2^{NF,mod})^2}{12}) \Rightarrow$$

$$\Rightarrow y \leq \hat{y} = \frac{b^2 - q^2 + \frac{1}{12}(z_3^{mod} + z_2^{NF,mod})}{2(b-q)+1}$$

if $2(b-q)+1 \leq 0$.

Proof of Proposition 9: To prove the first part of Proposition 9, one simply has to note that when the extremist faction is more ideologically exclusive ($z_1^{ext} \leq z_1^{NF,ext}$), fewer incumbents vote for the status quo. Districts that prefer the bill ($y \in [\frac{b+q}{2}, 1]$) will accrue greater welfare whereas those that prefer the status quo ($y \in [-1, \frac{b+q}{2}]$) will not.

To prove the second part of Proposition 9, starting with the case of $\frac{b+q}{2} \geq 0$, it is clear that all constituencies $y \leq 0$ benefit from the factions equilibrium, since fewer incumbents are voting for the status quo.

For the case of $\frac{b+q}{2} \leq 0$, district y benefits from the factions equilibrium if

$$((y-q)^2 - (y-b)^2)(z_3^{mod} - z_2^{mod})(-y_2^{mod}) - (y - \frac{z_3^{mod}}{2})^2 - \frac{(z_3^{mod})^2}{12} - (-(y-\frac{z_2^{mod}}{2})^2 - \frac{(z_2^{NF,mod})^2}{12})$$

which is equivalent to $y \leq \frac{\frac{13}{12}(z_3^{mod}+z_2^{mod})+y_2^{mod}(q^2-b^2)}{2((b-q)(-y_2^{mod})+1)} = \bar{y}$.

Proof of Proposition 10: The full equilibrium can be described as follows:

- For districts $y \in [0, y_1^{min}]$, members $z > z_1^{min}$ do not join a faction and vote for the status quo. Members $z \in [0, z_1^{min}]$ do join a faction and vote the majority party's bill.
- For districts $y \in [y_1^{min}, y_2^{min})$, when $c \geq 2b - b^2$ or $b \leq 0$, members $z \geq z_2^{min}$ vote for the status quo, and members $z < z_2^{min}$ vote for the majority party's bill. Those in the former category do not get reelected, and those in the latter category do. When $b \geq 0$ and $c \leq 2b - b^2$, all legislators vote in favor of the majority party's bill.
- Finally, in districts $y \in [y_2^{min}, 1]$, when $c \leq 1 - b^2$, members $z \geq z_3^{min}$ vote for the status quo and members $z < z_3^{min}$ vote for the majority party's bill. All legislators in these districts get reelected. If $c \geq 1 - b^2$, then everyone votes for the status quo.

In this proof, $z_1^{min} = -(1-b) + x_f^0 + \sqrt{2(1-x_f^0)(1-b) + w}$, $z_2^{min} = \frac{b+q}{2} + \frac{w-c}{2(q-b)}$, $z_3^{min} = \frac{b+q}{2} - \frac{c}{2(q-b)}$, $y_1^{min} = \frac{1}{3} z_1^{min} \frac{1+z_1^{min}}{2+z_1^{min}}$, and $y_2^{min} = \frac{1}{3} z_3^{min} \frac{1+z_3^{min}}{2+z_3^{min}}$.

For districts $[y_2, 1]$, the voters' posterior mean is

$$\mu(z) = \begin{cases} \frac{1+z_3^{min}}{2} & f = 0 \text{ and } v = q \\ \frac{z_3^{min}}{2} & f = 0 \text{ and } v = b \\ -\frac{1}{2} & \text{otherwise} \end{cases}$$

and the voters' posterior variance is

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(1 - z_3^{min})^2 & f = 0 \text{ and } v = q \\ \frac{(z_3^{min})^2}{12} & f = 0 \text{ and } v = b \\ \frac{1}{12} & \text{otherwise} \end{cases}$$

The incumbents prefer to vote for the status quo when

$$U_{z,f=0,v=q} = -(z-q)^2 \geq -(z-b)^2 - c = U_{z,f=0,v=b}$$

which is equivalent to $z \geq \frac{b+q}{2} - \frac{c}{2(q-b)}$. There is no incentive to deviate to joining a faction for members who vote for the bill because

$$U_{z,f=0,v=b} = -(z-b)^2 + w - c \geq -(z-b)^2 - (z-x_f)^2 = U_{z,f=1,v=b}$$

which is equivalent to $w - c \geq -(z - x_f)^2$. There is no incentive for those voting for the status quo to join a faction and vote for the bill when

$$U_{z,f=0,v=q} = -(z-q)^2 + w \geq -(z-x_f)^2 - (z-b)^2 = U_{z,f=1,v=q}$$

which is never satisfied for $z \geq \frac{b+q}{2} - \frac{c}{2(q-b)}$.

If $c \leq \frac{b+1}{1-b} + 2(1-b)^2 - 2x_f^0(1-b) + w - 2(1-b)\sqrt{2(1-b)(1-x_f^0)+w}$ and $c \geq 2b - b^2$ or $b \leq 0$, for districts $[y_1^{min}, y_2^{min}]$, the voters' posterior mean is

$$\mu(z) = \begin{cases} \frac{1+z_2^{min}}{2} & f = 0 \text{ and } v = q \\ \frac{z_2^{min}}{2} & f = 0 \text{ and } v = b \\ -\frac{1}{2} & \text{otherwise} \end{cases}$$

and the voters' posterior variance is

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(1 - z_2^{min})^2 & f = 0 \text{ and } v = q \\ \frac{(z_2^{min})^2}{12} & f = 0 \text{ and } v = b \\ \frac{1}{12} & \text{otherwise} \end{cases}$$

Incumbents prefer to vote for the status quo when

$$U_{z,f=0,v=q} = -(z-q)^2 \geq -(z-b)^2 + w - c = U_{z,f=0,v=b}$$

which is equivalent to $z \geq \frac{w-c}{2} + \frac{b+q}{2}$.

Incumbents who vote for the bill prefer not to join a faction when

$$U_{z,f=0,v=b} = -(z-b)^2 + w - c \geq -(z-b)^2 - (z-x_f)^2 = U_{z,f=1,v=b}$$

which is always true. Incumbents who vote for the status quo have no incentive to join a faction and vote for the bill when

$$U_{z,f=0,v=q} = -(z-q)^2 \geq -(z-b)^2 - (z-x_f)^2 = U_{z,f=1,v=b}$$

which yields a threshold less than z_1^{min}, which is lower than z_2^{min}.

For districts $[0, y_1^{min}]$, the voters' posterior mean is

$$\mu(z) = \begin{cases} \frac{1+z_1^{min}}{2} & f = 0 \text{ and } v = q \\ \frac{z_1^{min}}{2} & f = 1 \text{ and } v = b \\ -\frac{1}{2} & \text{otherwise} \end{cases}$$

and the voters' posterior variance is

$$\sigma^2(z) = \begin{cases} \frac{1}{12}(1 - z_1^{min})^2 & f = 0 \text{ and } v = q \\ \frac{(z_2^{min})^1}{12} & f = 1 \text{ and } v = b \\ \frac{1}{12} & \text{otherwise} \end{cases}$$

Incumbents prefer to vote for the status quo when

$$U_{z,f=0,v=q} = -(z-q)^2 \geq -(z-x_f)^2 - (z-b)^2 + w = U_{z,f=1,v=b}$$

which is equivalent to $z \geq -(1-b) + x_f^0 + \sqrt{2(1-x_f^0)(1-b) + w} = z_1^{min}$. Note that this threshold is always larger than zero. Incumbents who join a faction have no incentive to vote for the status quo in the later stage when

$$-(1-x_f) + \sqrt{2(1-x_f) + w} \leq \frac{1}{2} + \frac{w}{2}$$

which is always true. Incumbents who do not join a faction do not have an incentive to vote for the bill when

$$-(1-x_f) + \sqrt{2(1-x_f) + w} \geq \frac{1}{2} - \frac{w}{2}$$

which is true when $\frac{w^2}{4} - w(\frac{5}{2} - x_f) + (x_f - \frac{1}{2})^2 \leq 0$.

If $c > 1 + 2(1-x_f) - 2\sqrt{2(1-x_f) + w}$, for districts $[y_1^{min}, y_2^{min}]$, the voters' posterior mean and posterior variance is the same as for districts $[y_2^{min}, 1]$.